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GUGGENHEIM AERONAUTICAL LABORATORY

CALIFORNIA INSTITUTE OF TECHNOLOGY

EXPERIMENTAL DEFLECTION SURVEY OF CANTILEVER SECTORS OF UNIFORM THICKNESS

Thesis by

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EXPERIMENTAL DEFLECTION SURVEY
OF CANTILEVER SECTORS OF UNIFORM THICKNESS

Thesis by

William E. Henry, Lieutenant Commander, U.S.N.

In Partial Fulfillment of the Requirements
For the Degree of
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The research was carried out in collaboration with Lieutenant Commander Joseph Garrett, U.S.N.

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EXPERIMENTAL DEFLECTION SURVEY OF CANTILEVER SECTORS OF UNIFORM THICKNESS

SUMMARY

The purpose of this investigation was to study the deflection patterns of uniform thickness 24ST aluminum alloy sectors fixed on one radius and subjected to transverse loads.

This investigation consisted of obtaining deflection data in the form of influence coefficients for cantilever sectors of opening angles varying from 0 to 180 degrees.

The deflection data is presented in a form that requires only a matrix multiplication to obtain the deflection pattern of any sector of the same material constants and dimensions as those used in this investigation. For sectors of different material constants and dimensions the deflection pattern may be obtained by interpolation of the data presented in this investigation and use of elementary elastic relationships.

The comparison of the experimental and analytical solution of the deflection pattern of a 45 degree sector subjected to a shear loading and radial moments along the curved boundary showed satisfactory agreement.

This investigation was carried out at the Guggenheim Aeronautical Laboratory, California Institute of Technology, Pasadena, California (referred to hereafter as GALCIT).

EXPERIMENTAL DEFLECTION SURVEY
OF CANTILEVER SECTORS OF UNIFORM THICKNESS

I INTRODUCTION

The purpose of this investigation was to study the deflection of uniform thickness sectors fixed on one radius and subjected to tranverse loadings. All specimens used in this investigation were 24ST aluminum alloy. The stresses due to all loadings were below the proportional limit of the material.

The experimental work was divided into two phases:

Phase 1. Deflection surveys of a family of sectors of varying opening angles.

All sectors were of identical thickness and radius.

The deflection data obtained in this phase were converted into influence coefficients that are presented in matrix form in the tables of this report.

Phase 2. Deflection survey of a 45 degree sector subjected to a particular boundary loading.

The purpose of this phase was to obtain results that could be compared with an analytical solution. (See Reference 1).

A preliminary investigation of the effect of plate thickness on the deflection pattern of the specimens was included in this phase.

The testing equipment was designed and constructed by the author in collaboration with Joseph Garrett. The basic facilities of the GALCIT structures laboratory were used to advantage in the construction of the testing equipment.

Considerable time was spent in the development of procedures and techniques which would permit the investigation to proceed more rapidly yet yield accurate information on the deflection patterns of cantilever sectors.

II EQUIPMENT

Test Specimens

Phase 1. The original test specimen was 19.94 inches in radius with an average thickness of 0.251 inches and an opening angle of 180 degrees. Test specimens with opening angles of 135, 90, 75, 60, 45, and 30 degrees were obtained by progressively cutting back the original specimen. Figure 1 shows the plan form of the original specimen and the extension that was used to support the specimen in the testing equipment.

Two inch by 15 degree polar grids were scribed on each side of the original test specimen after it had been painted with a light coat of "Dykem blue".

Phase 2. The test specimen for this phase was 25 inches in radius with an average thickness of 0.125 inches and an opening angle of 45 degrees. Radial saw cuts made between the 25 inch radius and a 20 inch radius gave the specimen an "effective" radius of 20 inches. The saw cuts were made at 1 inch intervals of arc along the "effective" circumference. The strips formed by these saw cuts were used to apply radial moments along the "effective" circumference. Figure 2 shows the plan form of the Phase 2 test specimen.

Testing Equipment

The testing equipment was constructed using an existing heavy steel frame as a base. Two 2-1/2 x 4 x 3/4 inch angle sections approximately 43 inches long were leveled and secured to the existing base frame. The upper horizontal surface of the angle irons had been machined to provide a level surface for the hold-down plates. Two stress relieved and machined 1 x 29 x 19-1/2 inch hold-down plates were secured to the angle irons by 12 steel bolts. The specimen was inserted between the hold-down plates and shims 1/32 inch thinner than the specimen were used to prevent excessive "bowing" of the hold-down plates. Since hold-down bolts could not be used near the line of fixity of the specimen, three screw jacks were employed to increase the degree of fixity in this area. Figure 3 shows the arrangement used to secure the specimen.

The loading arrangement permitted the application of point loads from above to any point on the specimen by the use of a loading pin to which weights could be attached. The point of the loading pin was ground to as small a radius as possible without its causing damage to the specimen during loading.

The movement of the loading pin to the various grid points was accomplished in the following manner:

(a) The loading pin was raised from contact with the specimen by a four foot lever arm that had a fulcrum above the sector center.

(b) The lever arm was free to rotate about the sector center throughout the required 180 degrees carrying the loading pin with it.

(c) Radial motion of the loading pin was accomplished by means of rollers on the guiding mechanism. These rollers acted on the lever arm and could be locked at any radial position.

(d) The loading pin guiding mechanism was so arranged that when the load was positioned on the specimen neither the guiding mechanism nor the lever arm supported any of the load.

For further information on the loading see the enclosed figures.

A deflection table was positioned parallel and 9-1/2 inches below the lower surface of the test specimen. This deflection table consisted of an ordinary office table that was secured to the testing frame by means of "c" clamps. A 30 x 60 inch smooth surface top was constructed from 1/4 inch masonite glued to 1 inch plywood. When rigidly clamped to the office table this top provided a smooth, steady platform from which the deflections of the test specimen could be measured.

A deflection gauge was constructed by mounting a dial gauge of 1 inch travel reading to .001 of an inch on a steel base. The main spring of the dial gauge was removed and a uniform dial gauge force was obtained by gravity action on a horizontal 8 inch aluminum bar supported at an off-center pivot. The overall height of the dial gauge was made adjustable by the addition of precision ground base blocks.

Figure 4 illustrates the operation of the deflection gauge.

III INITIAL CALIBRATION AND PRELIMINARY TESTING

In order to determine the most effective experimental techniques a series of preliminary tests were conducted on a .075 inch 24ST aluminum apecimen of 20 inch radius. As a result of these tests it was found that the specimen should be gravity loaded from above and the deflections measured from below.

The following general requirements were set forth:

(a) The magnitude of the loads must be large enough to give a maximum deflection of about 1 inch providing that permanent "set" of the specimen does not occur.

(b) Sector angles between 0 and 180 degrees should be investigated.

(c) The deflection data obtained must be of such a nature that deflection patterns of the test specimen due to any transverse loading can be determined.

The general requirements indicated that the superposition of the deflections due to point loads and the use of Maxwell's Reciprocal Theorem would give the desired general deflection information with a minimum amount of data required.

The preliminary tests utilized a standard spring-loaded dial gauge. The results obtained did not satisfy Maxwell's Reciprocal Theorem. The following were considered as possible causes for the discrepancy.

(a) The fixity of the specimen in the testing equipment.

(b) Linear variation in the deflection gauge force on the specimen because of the main spring of the dial gauge.

By replacing the main spring of the dial gauge with a lever system that produced a uniform deflection gauge force throughout its range of travel, the results satisfied Maxwell's Reciprocal Theorem.

It was found that the most accurate results were obtained by leaving the deflection gauge under a given point while the load was moved from point to point. This procedure made it possible to check the "tare" reading each time the load was moved, and therefore read the deflections directly.

IV EXPERIMENTAL PROCEDURE

Phase 1.

The 180 degree sector was the initial test specimen. The specimen was secured in the testing equipment and random deflection readings were taken to insure that the data would satisfy Maxwell's Reciprocal Theorem. Fifty-three grid stations were selected as test points. The deflection gauge was placed under a test point and the dial indicator set on zero with no load applied. Weights had been attached to the loading pin until the total weight of the loading pins and weights was 50 pounds. The 50 pound load was moved to all test points. Deflection readings were multiplied by 20 and recorded on the data sheets. The numbers recorded therefore represented inches deflection per 1000 pounds. These recorded numbers are hereafter referred to as influence coefficients and designated "g_{ij}".

Maxwell's Reciprocal Theorem states that g_{ij} is equal to g_{ji}. This means that the deflection at a point "i" due to a load at a point "j" is equal to the deflection at point "j" due to the same load at "i". Applying this principle to the problem at hand permits a point to be disregarded as a loading point once the deflection gauge has been positioned under it and all deflection data recorded.

The deflection data obtained are presented as matrices of Influence Coefficients in Tables 1 through 7.

Contour lines drawn on the data sheets provided a rough check on the deflection data.

After a complete check of the deflection data for the 180 degree sector, the specimen was removed, cut back to 135 degrees, and the above testing procedure was repeated.

Similar experimental procedure was followed in the deflection surveys on the 90, 75, 60, 45, and 30 degree sectors. The test points used for each specimen are indicated in Tables 1 through 7.

Phase 2.

The method of measuring deflections in this phase was the same as Phase 1.

The specimen was loaded with a specific distributed boundary loading of -49.9 inch pounds radial moment per inch and -10.4 pounds shear per inch along the "effective circumference". This was accomplished by placing a 10.4 pound load 4.8 inches from the root of each of the radial strips.

Influence coefficients for shear alone were obtained by placing a 10 pound concentrated load at the root of each radial strip.

The influence coefficients for the radial moment alone were calculated from the data by the method shown in the Appendix to this report. Tables 8a and 8b give the influence coefficients for this phase.

A preliminary investigation of the relative stiffness of the 1/4 and 1/8 inch plates used in the two phases was made by comparing the elements of corresponding matrices of influence coefficients for 5 points on the free boundary. Table 9 shows a comparison of the stiffness of the two plates.

V CASE OF DISTRIBUTED LOADING

The deflection pattern of a circular sector due to a distributed loading can be computed by using the influence coefficients presented in this investigation if an area coefficient is associated with each of the loading points indicated in the matrices of influence coefficients.

The area coefficients must satisfy the following requirements:

(1) $P_j = a_j q_j$ Where "q_j" is the intensity of the distributed loading at the load point "j", and "a_j" is the area coefficient for the point "j".

(2) $w_i = \sum_{j=1}^{n} a_j q(r,\theta) g_{ij}$ Where $q(r,\theta)$ is determined from (1).

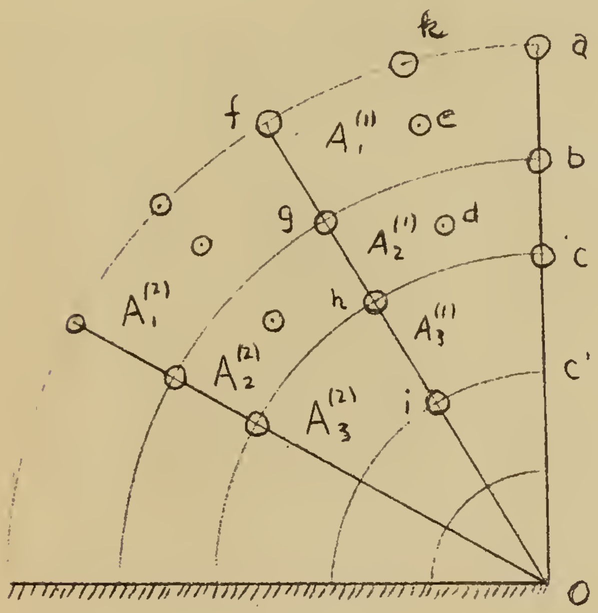
$= \sum_{j=1}^{n} P_j g_{ij}$ For all deflection points "i"

As indicated by the symmetrical matrices in Tables 1 through 7, there are the same number of deflection points, "i", as there are loading points, "j".

To meet the above requirements the area, "a_j" depends on: The relative geometric position of the load point "j" to the other load points and the plate boundaries, the load distribution as defined by $q(r,\theta)$, and the deflection mode near the deflection point "i". This means that "a_j" cannot be uniquely defined for all values of load distribution and still satisfy the above requirements, but for a limited class of load distributions a unique "a_j" can be assigned each load point that will satisfy the requirements to an acceptable degree of accuracy.

The load distribution has small variations over area "a_j" and as a first approximation a linear variation of influence coefficients was used.

The non symmetrical distribution of load points in Phase 1 of this experiment made it impossible to establish a general equation for the determination of all area coefficients. The following is an example of the method used to assign values of "a_j" to each load point:



o - Load points

The area $A_1^{(1)}$ is bounded by or contains the load points a, b, g, f, k, and e. The area $A_1^{(1)}$ was distributed to these load points in the following manner:

$\frac{A_1^{(1)}}{8}$ to a, b, g, and f,

$\frac{A_1^{(1)}}{2}$ to e,

$0A_1^{(1)}$ to k.

The assignment of a zero value of area to "k" was made because of its lack of symmetry in the pattern of the points. This does not eliminate point "k" as a deflection point.

The assignment of the value $\frac{A_1^{(1)}}{2}$ is justified by examining the load point "g" which would be assigned one eighth of the value of each of the adjacent areas. The typical area coefficients are:

$$a_a = \frac{A_1^{(1)}}{8} \qquad a_f = \frac{A_1}{4} \qquad a_e = \frac{A_1}{2}$$

$$a_k = 0 \qquad a_g = \frac{A_1}{4} + \frac{A_2}{4} \qquad a_b = \frac{A_1}{8} + \frac{A_2}{8}$$

For the area c, h, i, c' where c' is not a loading point, the area $A_3^{(1)}$ is first distributed equally to the 4 corner points, and then the area a'_c is redistributed to point "c" and the point "o" inversely as the distance to those points.

The deflection of any point becomes:

$$w_i = a_a q_a g_{ia} + a_b q_b g_{ib} + \ldots a'_c q_c g_{ic} + a_{c'} q_{c'} g_{ic'} + \ldots$$

$$\ldots + a_o q_o g_{io}$$

Where the last term vanishes since $g_{io} = 0$

Let $w_i = a'_c q_c g_{ic} \quad a_{c'} a_{c'} g_{ic'} < w_i$

If g_{ij} varies linearly and $q_{c'} = q_c = q_o$

Then $w_i = a'_c q_c g_{ic} + a_{c'} q_{c'} \frac{2}{3} g_{ic} = a_c q_c g_{ic}$

Or $(a'_c + \frac{2}{3} a_{c'}) q_c g_{ic} = a_c q_c g_{ic}$

And $a'_c + \frac{2a}{3}_{c'} = a_c$ where the area a'_c would be the area coefficient of the point c if c' were a load point.

This justifies the method of assigning values of area assumed by non-load points to other load points inversely as the distance to those points.

The values of the area coefficients assigned to each load point of the sectors investigated are tabulated in Tables 10 and 11. The area coefficients are for a sector of 20 inch radius and a multiplication by $r^2/400$ is required to make them applicable to sectors of other radii. Tables 10 and 11 give the area coefficients in column matrix form, and for convenience of notation the column matrix is designated $[A]_\alpha$ where α is the sector angle of the test specimen. The arrangement of the column matrix corresponds with the order of the associated influence coefficient matrix.

VI RESULTS AND DISCUSSION

Phase 1.

The results of this phase of the investigation are tabulated in the matrices of influence coefficients in Tables 1 through 7, and in the matrices of area coefficients in Tables 10 and 11. These results as presented apply only to cantilever sectors of 24 ST aluminum alloy, 20 inches in radius and 1/4 inch thick. The loadings must give rise to stresses within the proportional limit of the material. The deflection pattern of any sector with the above dimensions can be determined by proper use of the influence and area coefficients. The method of applying this data to specimens with different radius, thickness, or material constants will be described later.

The deflection produced by a concentrated load "P" at any loading point "j" is given by

$$w_i = P\, g_{ij} \times 10^{-3} \qquad (1)$$

The possible sources of error in the tabulated influence coefficients are:

(1) Method used in measuring deflections.

(2) Degree of fixity at the "built in" radius.

(3) Method of loading the test specimen.

(4) Imperfections in the test specimen.

The only method of checking the percent of error induced by the method of experimentation was to take repeated test readings on various points and apply Maxwell's Reciprocal Theorem to the resulting data. The error due to the methods of loading and measuring deflections was estimated to be ± .5% or ± .02. It was beyond the scope of this investigation to evaluate the error induced by the degree of fixity and possible imperfections in the material, but it is believed to be of the same magnitude as the error due to experimentation.

The deflection at grid points due to concentrated loads at points other than loading points would require interpolation. The accuracy of the resulting deflection data would depend on the accuracy of the interpolation. Graphical interpolation would be a desirable method of obtaining values of deflection at points other than the deflection points used in this investigation.

For a continuous shear loading along the curved boundary, the deflection at any point is given by:

$$w_i = \int_c V_{(s)}\, g_{i(s)}\, ds \qquad (2)$$

The following discussion gives the method of determining the deflection pattern due to a distributed load of intensity "q_j" at the grid point "j".

The deflection is given by:

$$w_i = \sum_{j=1} q_j a_j g_{ij} \times 10^{-3} \qquad (3)$$

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The equation in matrix form is:

$$\left[\;G\;\right]_{\alpha}\left[Q\right]_{\alpha} = \left[W\right]_{\alpha} \;\; x\;10^{-3} \qquad (4)$$

Where

$\left[\;G\;\right]$ is the square matrix of influence coefficients

$\left[Q\right]$ is the column matrix of area coefficients

$\left[W\right]$ is the column matrix of the elements "w_i" of the deflection

α is the sector angle.

For sectors of thickness, radii, or material constants different from those used in this investigation, the deflections can be obtained by the use of elasticity relationships. These relationships are outlined in the appendix to this report (see Reference 2).

Phase 2.

The results of this phase are tabulated in Tables 8a and 8b. The influence coefficient for the shear loading is designated by ${}_{v}G_{ij}$. This corresponds to the deflection in inches at "i" due to 1000 pounds of shear per inch acting over 1 inch of arc at "j". The influence coefficient for the radial moment is designated ${}_{m}g_{ij}$. This corresponds to the deflection at "i" due to a 1000 inch pound moment acting over 1 inch of arc at "j". The influence coefficient for the concentrated load is designated ${}_{c}g_{ij}$. This corresponds to the deflection at "i" due to a 1000 pound concentrated load at "j".

By superposition of deflections, the deflection pattern of the sector can be obtained by:

$$w_i = \left[\sum_{j=1}^{15} V_j \; {}_{v}g_{ij} + .71 V_{16} \; {}_{v}g_{i16} + \sum_{j=1}^{15} M_j \; {}_{m}g_{ij} + .71 M_{16} \; {}_{m}g_{i16} + P \; {}_{c}g_{cij} \right] x \; 10^{-3} \qquad (5)$$

The influence coefficients in Tables 8a and 8b are applicable to 45 degree sectors with the following properties:

(a) 24ST aluminum alloy

(b) Radius of 20 inches

(c) 1/8 inch thick.

For 45 degree sectors with different properties from the above, the deflection can be obtained by using elasticity relationships for thin plates (see appendix).

The deflections at 6 points on the free boundary were computed for three specific boundary loadings. These boundary loadings are shown in Figures 6, 7, and 8. The deflections were calculated by using equation (5). These deflections are compared with an analytical solution obtained by Mr. M. L. Williams (cf Ref 1).

For a comparison of the deflections of the analytical and experimental solutions see Figures 9, 10, and 11. The following observations are made:

1. The maximum deflections are of greater magnitude in the analytical solution for all three loading conditions.

2. The deflections of both solutions are of the same order of magnitude.

3. The deflection modes are very similar which indicates that the stresses should be in good agreement.

A preliminary investigation was made of the effect of the thickness of sector plates on their stiffness. A comparison of the influence coefficients of the corresponding sectors in Phase 1 and Phase 2 is shown in Table 9. The radial strips on the Phase 2 sector were removed for this investigation. Elasticity relations show that the deflections of plates of different thickness vary inversely as the plate stiffness. If the plates have the same material constants, the deflections will vary inversely as the thickness cubed, (t^3). The ratio of the analytical deflection of the .125 inch plate to the .251 inch plate is:

$$\frac{\text{Deflection of .125 plate}}{\text{Deflection of .251 plate}} = \frac{D(.251)}{D(.125)}$$

$$\text{Where} \quad D = \frac{Et^3}{12(1-V^2)}$$

$$\frac{D(.251)}{D(.125)} = \left[\frac{t(.251)}{t(.125)}\right]^3 = \left[\frac{.251}{.125}\right]^3 = 8.12$$

For the 5 points investigated, the experimental deflection ratio gives an average value of 7.30.

The possible causes of this variation are:

1. The material constants of two sectors may have been different.

2. The 1/8 inch sector may have permitted a higher degree of fixity at the "built in" radius.

VII CONCLUSIONS

The following conclusions are obtained from this investigation:

1. The use of influence coefficients is a very satisfactory method of obtaining general deflection information for cantilever sectors.

2. Comparison of the deflections obtained by experimental investigation with the deflections obtained by analytical investigation indicated that the experimental deflections were less than the analytical deflections.

3. Further deflection investigations should be performed to obtain more conclusive results on the effect of plate thickness on the deflection pattern.

VIII RECOMMENDATIONS

1. Various sectors should be equipped with strain gauges to determine the stresses in the specimen due to specific loading patterns. These stresses should be compared with stresses calculated from the experimental and theoritical deflection data obtained in previous investigations.

2. An extensive investigation should be made of the effect of plate thickness on stiffness.

REFERENCES

(1) Williams, M. L. "The Plate Problem for a Cantilever Sector of Uniform Thickness". Doctor's Thesis, California Institute of Technology.

(2) Sechler, E. E. "Elasticity in Engineering", Chapter XII, California Institute of Technology.

TABLES

MATRIX OF INFLUENCE COEFFICIENTS*
FOR 90 DEGREE SECTOR

Radius - 20 inches
Ave. Thickness - .251 inches
24 ST Aluminum Plate

G.	IN.	15/6	15/10	15/14	15/20	30/12	30/16	30/20	45/6	45/10	45/14	45/18	45/20	60/8	60/12	60/16	60/20	75/10	75/14	75/18	75/20	90/4	90/6	90/8	90/10	90/12	90/14	90/16	90/18	90/20
5	6	.02																												
	10	.02	.05																											
	14	.01	.04	.10																										
	20	.01	.04	.12	.52																									
0	12	.03	.10	.14	.16	.33																								
	16	.03	.11	.23	.38	.44	.80																							
	20	.04	.12	.31	.71	.54	1.12	1.92																						
5	8	.04	.05	.04	.04	.14	.16	.16	.18																					
	10	.05	.10	.13	.13	.37	.46	.54	.26	.54																				
	14	.06	.14	.24	.32	.60	.90	1.14	.30	.76	1.30																			
	18	.07	.18	.36	.56	.78	1.33	1.91	.34	.94	1.70	2.80																		
	20	.08	.20	.41	.68	.87	1.56	2.31	.17	1.03	2.02	4.29	4.00																	
0	8	.06	.09	.10	.08	.31	.36	.38	.32	.53	.68	.80	.84	.64																
	12	.08	.14	.20	.22	.58	.78	.90	.40	.86	1.33	1.72	1.88	.89	1.61															
	16	.10	.20	.32	.40	.83	1.24	1.60	.50	1.18	2.00	2.81	3.20	1.40	2.25	3.50														
	20	.11	.25	.44	.60	1.10	1.74	2.30	.58	1.47	2.64	4.00	4.66	1.73	2.86	4.70	6.77													
5	10	.08	.12	.14	.11	.46	.56	.60	.44	.80	1.09	1.30	1.42	.96	1.52	1.97	2.38	1.68												
	14	.10	.18	.25	.24	.76	1.01	1.16	.58	1.21	1.83	2.37	2.62	1.36	2.39	3.37	4.29	2.38	3.82											
	18	.13	.24	.37	.40	1.03	1.45	1.80	.72	1.59	2.58	3.52	3.95	1.67	3.16	4.82	6.48	2.98	5.14	7.43										
	20	.14	.27	.42	.49	1.17	1.69	2.08	.76	1.76	2.93	4.09	4.63	1.81	3.56	5.53	7.58	3.28	5.77	8.60	10.09									
0	4	.02	.03	.02	.00	.08	.07	.05	.16	.20	.20	.20	.18	.32	.35	.38	.40	.47	.56	.60	.64	.35								
	6	.04	.06	.04	.00	.18	.18	.14	.28	.40	.44	.45	.45	.59	.71	.81	.88	.93	1.14	1.33	1.38	.47	.82							
	8	.06	.08	.08	.03	.32	.34	.29	.40	.61	.74	.80	.82	.86	1.16	1.14	1.56	1.44	1.88	2.21	2.38	.55	1.04	1.52						
	10	.08	.12	.12	.06	.44	.50	.47	.48	.83	1.08	1.22	1.27	1.07	1.61	2.03	2.37	1.94	2.68	3.26	3.53	.61	1.20	1.86	2.48					
	12	.10	.15	.17	.10	.58	.89	.70	.58	1.06	1.44	1.70	1.80	1.32	2.09	2.75	3.29	2.41	3.56	4.42	4.88	.67	1.34	2.16	2.98	3.80				
	14	.11	.18	.22	.16	.73	.90	.94	.67	1.29	1.82	2.22	2.37	1.53	2.57	3.49	4.28	2.85	4.38	5.72	6.32	.72	1.51	2.42	3.43	4.50	5.54			
	16	.13	.21	.27	.21	.88	1.09	1.20	.75	1.50	2.20	2.74	2.98	1.76	3.04	4.28	5.34	3.23	5.16	7.01	7.88	.76	1.61	2.66	3.84	5.10	6.45	7.78		
	18	.14	24	.32	.27	1.02	1.29	1.47	.82	1.72	2.56	3.29	3.60	2.88	3.47	4.97	6.40	5.61	5.95	8.30	9.46	.84	1.76	1.95	4.22	5.67	7.27	8.94	10.54	
	20	.16	.28	.36	.30	1.15	1.52	1.70	.89	1.90	2.92	3.83	4.23	2.13	3.91	5.76	7.52	4.00	6.72	9.84	11.08	.89	1.88	3.16	4.60	6.26	8.07	10.09	12.10	14.18

(Inches deflection per pound) x10^3

TABLE 3

MATRIX OF INFLUENCE COEFFICIENTS*
FOR 180 DEGREE SECTOR

Radius - 20 inches
Ave. Thickness - .251 inches
24 ST Aluminum Plate

DEG.	IN.	15/6	15/10	15/14	15/20	30/12	30/16	30/20	45/6	45/10	45/14	45/18	45/20	60/8	60/12	60/16	60/20	75/10	75/14	75/18	75/20	90/4	90/8	90/12	90/16	90/20	105/6	105/10	105/14	105/18	105/20	120/8	120/12	120/16	120/20	135/10	135/14	135/18	135/20	150/4	150/8	150/12	150/16	150/20	165/6	165/10	165/14	165/18	165/20	180/12	180/14	180/16	180/18	180/20
15	6	.03																																																				
	10	.01	.06																																																			
	14	.02	.04	.10																																																	
	20	.01	.02	.12	.51																																															
30	12	.04	.08	.15	.15	.31																																														
	16	.03	.09	.21	.35	.40	.73																																													
	20	.04	.10	.28	.66	.49	1.02	1.79																																												
45	6	.05	.04	.06	.04	.16	.16	.18	.17																																										
	10	.05	.09	.12	.12	.34	.41	.48	.25	.49																																									
	14	.08	.13	.22	.28	.54	.78	1.02	.28	.68	1.16																																								
	18	.07	.16	.32	.51	.71	1.20	1.74	.35	.84	1.58	2.52																																							
	20	.05	.18	.34	.62	.75	1.41	2.08	.38	.89	1.78	2.90	3.58																																						
60	8	.05	.07	.09	.08	.28	.34	.38	.28	.47	.61	.74	.78	.54																																					
	12	.07	.13	.19	.17	.52	.69	.80	.37	.76	1.14	1.50	1.64	.80	1.40																																				
	16	.08	.17	.27	.34	.72	1.10	1.37	.45	1.02	1.73	2.44	2.76	1.00	1.89	2.94																																			
	20	.10	.22	.38	.52	.96	1.48	2.00	.55	1.28	2.28	3.39	3.95	1.23	2.43	3.91	5.68																																		
75	10	.06	.11	.14	.12	.42	.50	.58	.37	.68	.94	1.19	1.27	.80	1.28	1.68	2.09	1.35																																				
	14	.08	.16	.21	.22	.64	.84	.99	.52	1.00	1.54	2.02	2.24	1.12	1.93	2.79	3.60	1.92	3.07																																		
	18	.11	.20	.31	.33	.89	1.23	1.48	.64	1.31	2.13	2.96	3.28	1.42	2.60	3.93	5.22	2.49	4.14	5.88																																		
	20	.13	.24	.34	.38	.98	1.40	1.72	.68	1.49	2.40	3.35	3.76	1.59	2.91	4.51	6.11	2.77	4.62	6.77	7.92																																	
90	4	.04	.02	.03	.01	.11	.10	.10	.16	.20	.22	.27	.25	.28	.36	.44	.52	.44	.58	.72	.78	.25																																
	8	.05	.07	.08	.06	.28	.34	.33	.33	.52	.70	.81	.85	.66	.99	1.30	1.50	1.15	1.60	2.00	2.20	.46	1.11																															
	12	.09	.13	.17	.12	.50	.61	.65	.48	.86	1.21	1.51	1.61	1.03	1.70	2.27	2.83	1.86	2.75	3.81	4.01	.65	1.67	2.81																														
	16	.10	.17	.22	.19	.70	.92	1.01	.63	1.19	1.76	2.26	2.46	1.40	2.41	3.38	4.31	2.54	3.95	5.43	6.04	.83	2.24	3.91	5.68																													
	20	.15	.22	.29	.27	.91	1.21	1.36	.78	1.53	2.32	3.10	3.33	1.76	3.03	4.44	5.76	3.22	5.15	7.24	8.24	1.02	2.74	4.88	7.48	10.15																												
105	6	.00	.04	.04	.01	.17	.18	.16	.23	.33	.42	.54	.47	.47	.64	.78	.89	.80	1.06	1.30	1.42	.40	.86	1.20	1.60	1.90	.73																											
	10	.06	.08	.10	.04	.34	.34	.36	.39	.68	.83	.98	1.01	.84	1.29	1.63	1.97	1.56	2.15	2.76	2.99	.63	1.51	2.42	3.20	4.00	1.20	2.26																										
	14	.09	.13	.14	.08	.51	.62	.60	.54	.94	1.34	1.58	1.68	1.23	1.93	2.61	3.13	2.25	3.37	4.33	4.82	.85	2.16	3.65	5.04	6.38	1.68	3.26	4.96																									
	18	.12	.17	.20	.13	.71	.83	.83	.71	1.86	1.79	2.18	2.31	1.60	2.59	3.53	4.41	2.94	4.54	6.08	6.72	1.08	2.77	4.76	6.82	9.01	2.09	4.24	6.65	9.29																								
	20	.12	.18	.21	.12	.78	.94	.94	.78	1.42	1.99	2.48	2.64	1.78	2.90	4.04	4.96	3.30	5.06	6.87	7.07	1.18	3.10	5.30	7.79	10.37	2.34	4.66	7.52	10.53	12.41																							
120	8	.04	.05	.04	.00	.19	.19	.14	.28	.41	.50	.54	.54	.60	.83	1.02	1.15	1.09	1.44	1.77	1.92	.54	1.18	1.73	2.24	2.74	1.04	1.78	2.50	3.16	3.54	1.59																						
	12	.06	.07	.08	.00	.34	.33	.28	.43	.68	.84	.95	.95	.94	1.42	1.73	2.07	1.76	2.46	3.13	3.40	.78	1.83	2.92	3.94	4.86	1.53	2.92	4.28	5.56	6.11	2.40	3.98																					
	16	.08	.11	.10	.00	.46	.50	.41	.56	.93	1.21	1.38	1.40	1.31	1.98	2.54	2.99	2.46	3.51	4.54	4.98	1.03	2.52	4.07	5.66	7.13	2.04	3.95	6.04	8.06	9.10	3.24	5.49	7.95																				
	20	.12	.14	.14	.03	.60	.64	.57	.73	1.18	1.58	1.79	1.83	1.64	2.52	3.31	3.92	3.12	4.80	5.84	6.54	1.32	3.1[illegible]	5.20	7.43	9.48	2.54	4.99	7.76	10.73	12.13	4.01	7.05	10.27	13.92																			
135	10	.04	.04	.02	-.04	.17	.12	.04	.29	.40	.43	.44	.39	.64	.86	.97	1.10	1.20	1.57	1.93	2.03	.63	1.36	2.06	2.65	3.20	1.26	2.24	3.13	3.96	4.33	2.05	3.24	4.33	5.46	2.91																		
	14	.07	.06	.05	-.06	.30	.20	.10	.40	.71	.70	.82	.86	.92	1.28	1.58	1.71	1.78	2.47	2.98	3.27	1.02	1.98	3.09	4.10	5.03	1.79	3.27	4.84	6.23	6.92	2.90	4.81	6.74	8.55	4.15	6.37																	
	18	.07	.07	.06	-.06	.36	.27	.14	.54	.79	.95	.94	.88	1.22	1.74	2.11	2.38	2.36	3.32	4.10	4.43	1.14	2.63	4.16	5.62	6.94	2.31	4.33	6.47	8.62	9.60	3.79	6.42	9.18	11.97	5.48	8.46	11.90																
	20	.08	.08	.06	-.08	.38	.30	.12	.60	.90	1.04	1.07	1.02	1.36	1.95	2.45	2.66	2.67	3.62	4.62	4.98	1.28	2.90	4.84	6.34	7.89	2.34	4.84	7.30	9.79	10.94	4.22	7.17	10.48	13.70	6.02	9.56	13.36	15.36															
150	4	.02	.00	-.01	-.03	.01	-.02	-.06	.08	.06	.03	.00	-.05	.13	.15	.11	.08	.24	.27	.27	.28	.21	.34	.44	.50	.58	.38	.57	.70	.84	.87	.62	.83	1.04	1.27	.89	1.28	1.42	1.53	.44														
	8	.02	.01	-.01	-.06	.04	-.02	-.10	.16	.17	.14	.06	.00	.34	.38	.38	.33	.66	.80	.87	.90	.44	.88	1.19	1.44	1.67	.92	1.44	1.94	2.37	2.58	1.52	2.20	2.87	3.47	2.21	3.05	3.86	4.32	.80	1.91													
	12	.03	.01	-.01	-.09	.08	-.02	-.15	.25	.30	.24	.15	.05	.56	.70	.69	.67	1.12	1.37	1.60	1.62	.68	1.39	2.04	2.57	3.01	1.39	2.40	3.35	4.22	4.56	2.36	3.72	4.96	6.20	3.57	5.18	6.82	7.52	1.15	2.90	4.80												
	16	.04	.02	-.02	-.14	.11	-.02	-.20	.32	.42	.40	.23	.13	.77	1.00	1.05	.99	1.60	2.02	2.34	2.44	.91	1.95	2.91	3.70	4.41	1.86	3.38	4.80	6.13	6.72	3.23	5.24	7.24	9.15	4.91	7.43	9.98	11.15	1.46	3.84	6.62	9.58											
	20	.05	.03	-.03	-.15	.15	-.00	-.25	.44	.54	.49	.32	.17	1.00	1.27	1.34	1.32	2.02	2.63	3.04	3.18	1.15	2.47	3.90	4.90	5.81	2.38	4.28	6.21	8.06	8.85	4.07	6.78	9.44	12.10	6.25	9.61	13.25	14.97	1.78	4.77	8.44	12.45	16.77										
165	6	.01	-.01	-.03	-.07	-.04	-.10	-.16	.05	.01	-.06	-.15	-.20	.10	.06	.04	-.13	.23	.20	.14	.08	.24	.04	.47	.50	.49	.52	.72	.92	1.00	1.12	.88	1.17	1.46	1.67	1.33	1.76	2.16	3.28	.64	1.34	1.88	2.41	2.91	1.11									
	10	.00	-.01	-.04	-.10	-.06	-.16	-.28	.10	.04	-.08	-.24	-.33	.22	.17	.04	-.13	.50	.50	.44	.37	.46	.80	1.02	1.16	1.21	.93	1.45	1.90	2.24	2.35	1.86	2.41	3.07	3.70	2.68	3.66	4.58	4.98	1.03	2.43	3.80	5.03	6.28	1.82	3.47								
	14	.00	-.02	-.07	-.14	-.08	-.22	-.40	.14	.08	-.08	-.31	-.44	.32	.30	.14	-.11	.78	.83	.78	.72	.64	1.20	1.60	1.90	2.04	1.38	2.22	2.89	3.62	3.88	2.47	3.73	4.96	6.00	3.91	5.70	7.33	8.14	1.37	3.46	5.70	7.97	10.05	2.44	4.94	7.53							
	18	.01	-.03	-.09	-.20	-.09	-.29	-.53	.21	.13	-.07	-.38	-.57	.48	.42	.21	-.09	1.09	1.18	1.15	1.03	.86	1.63	2.21	2.67	2.98	1.80	3.00	4.08	5.07	5.43	3.21	5.05	6.77	8.42	5.18	7.71	10.30	11.43	1.76	4.46	7.67	10.95	14.16	3.06	8.30	10.13	13.85						
	20	.01	-.02	-.10	-.24	-.10	-.32	-.60	.22	.14	-.08	-.43	-.62	.53	.47	.26	-.10	1.21	1.36	1.28	1.24	.94	1.84	2.51	3.02	3.32	2.02	3.36	4.68	5.71	6.26	3.68	5.72	7.78	9.56	5.81	8.86	11.70	12.85	1.95	5.02	8.47	12.26	16.41	3.34	6.95	11.24	15.94	18.31					
180	12	-.02	-.05	-.10	-.14	-.29	-.34	-.49	-.01	-.17	-.39	-.65	-.80	.00	-.22	-.53	-.85	.12	-.09	-.43	-.58	.39	.53	.50	.39	.18	.83	1.40	1.31	1.38	1.33	1.57	2.76	2.86	3.00	2.74	3.74	4.53	4.92	1.22	2.81	4.30	5.64	6.88	2.26	4.29	6.17	7.86	8.68	6.00				
	14	.02	-.05	-.12	-.19	-.22	-.39	-.58	.00	-.18	-.45	-.67	-.93	.01	-.15	-.36	-1.02	.19	-.06	-.14	-.68	.44	.68	.66	.55	.26	.98	1.47	1.67	1.74	1.79	1.90	2.67	3.33	3.79	3.29	4.58	5.68	6.19	1.38	3.33	5.20	7.05	6.64	2.82	5.08	7.62	9.86	10.67	7.05	8.54			
	16	-.02	-.07	-.12	-.20	-.25	-.47	-.70	.00	-.20	-.53	-.90	-1.10	.03	-.28	-.68	-1.14	.23	-.06	-.26	-.71	.43	.77	.81	.67	.42	1.14	1.67	2.01	2.18	2.19	2.21	3.22	4.03	4.68	3.86	5.40	6.86	7.47	1.56	3.79	8.14	8.36	10.50	2.93	5.74	8.89	11.81	13.24	8.04	10.00	11.87		
	18	-.02	-.07	-.14	-.25	-.29	-.52	-.68	.00	-.23	-.57	-1.00	-1.24	.04	-.30	-.73	-1.30	.27	-.02	-.50	-.70	.58	.90	.94	.85	.53	1.30	1.91	2.38	2.58	2.71	2.54	3.69	4.68	5.44	4.40	6.34	7.99	8.89	1.74	4.29	7.01	9.30	13.30	3.22	6.55	10.18	13.84	15.64	9.01	11.32	13.56	15.87	
	20	-.02	-.08	-.17	-.25	-.32	-.58	-.87	.00	-.27	-.64	-1.10	-1.37	.05	-.30	-.49	-1.38	.31	-.03	-.47	-.79	.66	1.03	1.16	1.03	.78	1.47	2.22	2.74	3.07	3.18	2.86	4.25	5.31	6.41	4.98	7.14	9.24	10.19	1.90	4.82	7.94	11.13	14.32	3.54	7.30	11.51	15.90	18.10	10.08	12.58	15.32	17.99	21.06

TABLE 1

MATRIX OF INFLUENCE COEFFICIENTS*
FOR 135 DEGREE SECTOR

Radius - 20 inches
Ave. thickness - .251 inches
24 ST Aluminum Plate

DEG.	IN.																																											
15	6	.02																																										
	10	.01	.05																																									
	14	.01	.04	.10																																								
	20	.01	.03	.12	.50																																							
30	12	.03	.09	.13	.15	.31																																						
	16	.03	.10	.21	.36	.42	.75																																					
	20	.04	.11	.28	.65	.48	1.02	1.82																																				
45	6	.03	.04	.04	.04	.14	.16	.17	.17																																			
	10	.05	.09	.11	.12	.34	.43	.48	.23	.51																																		
	14	.06	.13	.22	.29	.55	.82	1.02	.31	.70	1.18																																	
	18	.07	.18	.31	.51	.72	1.25	1.74	.34	.86	1.64	2.54																																
	20	.07	.17	.38	.62	.80	1.44	2.12	.36	.98	1.84	3.02	3.69																															
60	8	.05	.08	.09	.08	.29	.35	.38	.28	.50	.65	0.75	.82	.58																														
	12	.07	.13	.18	.20	.54	.72	.84	.38	.79	1.22	1.57	1.73	.82	1.43																													
	16	.08	.17	.29	.35	.76	1.15	1.53	.46	1.05	1.80	2.53	2.89	1.08	2.02	3.08																												
	20	.10	.22	.38	.52	.98	1.54	2.05	.56	1.33	2.38	3.53	4.14	1.27	2.54	4.12	5.88																											
75	10	.06	.11	.13	.12	.43	.54	.58	.40	.73	1.01	1.26	1.36	.86	1.34	1.80	2.18	1.44																										
	14	.09	.14	.23	.23	.68	.89	1.04	.52	1.06	1.62	2.15	2.36	1.18	2.07	2.96	3.78	2.06	3.26																									
	18	.12	.21	.28	.34	.92	1.30	1.59	0.64	1.40	2.25	3.11	3.50	1.52	2.73	4.20	5.60	2.66	4.40	6.35																								
	20	.12	.23	.36	.42	1.04	1.51	1.86	.74	1.60	2.58	3.25	4.02	1.67	3.17	4.84	6.51	2.97	4.98	7.25	8.45																							
90	4	.02	.03	.02	.00	.10	.10	.09	.15	.18	.25	0.26	.26	.28	.38	.44	.49	.46	.59	0.72	.76	.27																						
	8	.05	.08	.08	.05	.30	.34	.34	.34	.53	.72	0.86	.88	.69	1.04	1.34	1.56	1.22	1.68	2.10	2.33	.49	1.18																					
	12	.08	.13	.16	.12	.53	.67	.68	.50	.92	1.30	1.62	1.74	1.14	1.81	2.48	2.99	2.00	2.97	3.65	4.35	.68	1.78	3.07																				
	16	.11	.17	.24	.20	.77	1.00	1.09	.66	1.26	1.92	2.44	2.69	1.52	2.60	3.66	4.60	2.76	4.30	5.86	6.74	.66	2.37	4.20	6.24																			
	20	.14	.22	.31	.29	.99	1.28	1.48	.83	1.62	2.46	3.28	3.62	1.87	3.30	4.82	6.29	3.46	5.57	7.88	9.06	1.02	2.95	5.38	8.14	11.13																		
105	6	.03	.04	.04	0	.14	.17	.14	.24	.34	.42	0.44	.45	.49	.66	.80	.90	.84	1.11	1.34	1.44	.44	.92	1.29	1.66	1.97	.81																	
	10	.06	.09	.09	.04	.36	.40	.36	.42	.70	.90	1.03	1.06	.92	1.34	1.76	2.06	1.66	2.30	2.90	3.23	.68	1.63	2.58	3.52	4.28	1.30	2.48																
	14	.09	.13	.15	.09	.56	.66	.63	.59	1.03	1.42	1.72	1.80	1.34	2.10	2.83	3.41	2.51	3.66	4.74	5.28	.88	2.32	3.92	5.52	7.10	1.71	3.57	5.62															
	18	.12	.17	.22	.14	.77	.93	.94	.72	1.40	1.94	2.37	2.58	1.74	2.86	3.87	4.85	3.28	4.98	6.68	7.47	1.12	3.05	5.22	7.61	10.01	2.24	4.64	7.42	10.38														
	20	.13	.19	.25	.16	.86	1.04	1.09	.84	1.57	2.20	2.72	2.97	1.90	3.25	4.44	5.51	3.66	5.62	7.72	8.60	1.22	3.36	5.94	8.70	11.50	2.42	5.16	8.38	11.99	13.70													
120	8	.04	.05	.04	-.02	.20	.18	.12	.30	.40	.52	0.52	.50	.68	.83	1.02	1.11	1.14	1.52	1.82	1.97	.60	1.26	1.84	2.38	2.85	1.16	1.94	2.73	3.40	3.70	1.81												
	12	.07	.08	.08	.01	.35	.36	.26	.47	.74	.90	1.00	1.02	1.05	1.47	1.87	2.15	3.95	2.72	3.34	3.70	.85	2.03	3.24	4.37	5.39	1.70	3.29	4.74	6.14	6.77	2.70	4.61											
	16	.09	.12	.12	.01	.51	.55	.45	.64	1.04	1.35	1.52	1.57	1.44	2.20	2.80	3.28	2.74	3.94	5.00	5.56	1.11	2.76	4.57	6.38	7.98	2.23	4.48	6.80	9.24	10.36	3.59	6.16	9.18										
	20	.12	.15	.16	.04	.67	.75	.85	.80	1.32	1.76	2.10	2.14	1.82	2.82	3.68	4.43	3.49	5.12	6.72	7.44	1.30	3.50	5.88	8.34	10.78	2.70	5.62	8.84	12.26	13.87	4.38	7.98	11.89	16.28									
135	4	.01	.01	0	-.03	.03	0	-.03	.08	.08	.10	0.05	.02	.17	.39	.20	.17	.31	.40	0.42	.42	.26	.43	.52	.64	.70	.46	.65	.84	0.97	1.23	.70	.92	1.18	1.36	.49								
	6	.02	.02	0	-.04	.07	.03	-.03	.15	.17	.20	0.14	.10	.33	.37	.43	.38	.62	.77	0.83	.87	.43	.78	1.04	1.26	1.42	.81	1.23	1.65	1.92	2.06	1.28	1.74	2.20	2.62	.70	1.14							
	8	.03	.03	.01	-.06	.11	.06	-.02	.24	.30	.28	0.26	.21	.51	.63	.70	.68	.96	1.20	1.36	1.42	.58	1.16	1.60	2.00	3.20	1.16	1.85	2.52	3.02	3.30	1.85	2.70	3.50	4.17	.65	1.50	2.15						
	10	.04	.04	.02	-.07	.16	.10	-.01	.30	.40	.45	0.40	.34	.68	.86	1.00	1.00	1.28	1.69	1.96	2.04	.72	1.51	2.22	2.84	3.29	1.42	2.48	3.48	4.32	4.68	2.42	3.64	4.91	5.93	.98	1.80	2.70	3.50					
	12	.05	.05	.03	-.08	.21	.15	.01	.37	.52	.60	0.54	.48	.86	1.14	1.32	1.39	1.64	2.18	2.60	2.76	.84	1.96	2.88	3.71	4.37	1.76	3.13	4.52	5.70	6.25	2.90	4.69	6.38	7.95	1.13	1.90	3.22	4.26	5.42				
	14	.06	.08	.04	-.08	.26	.21	.04	.47	.65	.76	0.71	.64	1.08	1.43	1.68	1.78	2.04	2.72	3.28	3.51	1.02	2.27	3.53	4.58	5.54	2.00	3.82	5.50	7.05	7.84	3.43	5.72	7.91	9.94	1.27	2.34	3.65	4.99	6.40	7.80			
	16	.07	.07	.05	-.08	.32	.26	.09	.56	.76	.88	0.88	.82	1.22	1.67	1.98	2.18	2.34	3.27	3.94	4.24	1.12	2.26	4.13	5.48	6.72	2.28	4.44	6.48	8.46	9.53	3.34	6.58	7.38	11.98	1.38	2.63	4.04	5.62	7.30	9.00	10.74		
	18	.08	.09	.06	-.08	.38	.32	.12	.60	.90	1.02	1.00	1.01	1.40	1.96	2.34	2.60	2.70	3.76	4.60	4.98	1.22	3.00	4.70	6.38	7.90	2.58	5.03	7.48	10.00	11.16	4.30	7.56	10.90	14.16	1.52	2.82	4.44	6.31	8.10	10.17	12.36	14.40	
	20	.08	.11	.06	-.08	.41	.33	.12	.66	1.01	1.17	1.17	1.16	1.56	2.22	2.73	3.02	3.05	4.24	5.27	5.78	1.37	3.34	5.38	7.29	9.10	2.78	5.64	8.56	11.54	12.82	4.72	8.44	12.36	16.41	1.59	3.07	4.86	6.97	9.06	11.62	14.04	16.66	19.21
DEG.	IN.	15/6	15/10	15/14	15/20	30/12	30/16	30/20	45/6	45/10	45/14	45/18	45/20	60/8	60/12	60/16	80.20	75/10	75/14	75/18	75/20	90/4	90/8	90/12	90/16	90/20	105/6	105/10	105/14	105/18	105/20	120/8	120/12	120/16	120/20	135/4	135/6	135/8	135/10	135/12	135/14	135/16	135/18	135/20

* (Inches deflection per pound) x10^3

TABLE 2

MATRIX OF INFLUENCE COEFFICIENTS*
FOR 75 DEGREE SECTOR

Radius - 20 inches
Ave. Thickness - .251 inches
24 ST Aluminum Plate

DEG.	IN.																							
15	6	.02																						
	10	.02	.04																					
	14	.01	.04	.12																				
	20	.01	.04	.13	.54																			
30	12	.04	.10	.15	.17	.36																		
	16	.03	.10	.24	.42	.46	.83																	
	20	.03	.11	.32	.74	.56	1.17	2.00																
45	6	.04	.04	.04	.03	.18	.16	.15	.20															
	10	.06	.10	.14	.14	.40	.47	.54	.28	.58														
	14	.06	.14	.26	.33	.62	.94	1.19	.31	.80	1.39													
	18	.07	.18	.38	.60	.82	1.42	2.02	.34	.98	1.89	2.96												
	20	.08	.20	.43	.72	.91	1.64	2.42	.35	1.04	2.12	3.48	4.22											
60	8	.06	.09	.10	.07	.33	.36	.36	.33	.57	.70	.80	.82	.70										
	12	.08	.15	.22	.22	.63	.87	.93	.44	.95	1.42	1.80	1.94	.98	1.67									
	16	.10	.20	.35	.42	.91	1.35	1.69	.51	1.26	2.16	3.01	3.38	1.20	2.43	3.80								
	20	.12	.25	.46	.66	1.14	1.85	2.44	.55	1.52	2.84	4.28	4.99	1.40	2.98	5.04	7.34							
75	4	.08	.02	.02	.00	.09	.07	.04	.17	.20	.20	.18	.16	.30	.34	.35	.34	.27						
	6	.06	.06	.05	.02	.20	.20	.16	.30	.41	.44	.46	.44	.58	.72	.78	.84	.36	.64					
	8	.08	.10	.10	.05	.35	.36	.33	.41	.65	.78	.84	.85	.87	1.18	1.39	1.54	.40	.80	1.20				
	10	.09	.13	.15	.10	.51	.57	.56	.50	.90	1.16	1.34	1.37	1.10	1.69	2.10	2.41	.44	.90	1.43	1.95			
	12	.10	.16	.21	.17	.65	.80	.86	.55	1.10	1.57	1.90	2.00	1.26	2.14	2.89	3.44	.45	.98	1.61	2.30	2.94		
	14	.12	.19	.27	.24	.82	1.07	1.19	.62	1.33	.99	2.53	2.70	1.44	1.64	3.72	4.58	.48	1.06	1.80	2.63	3.52	4.36	
	16	.13	.22	.34	.33	.98	1.34	1.54	.67	1.52	2.40	3.17	3.47	1.60	3.05	4.53	5.80	.50	1.12	1.94	2.89	3.96	5.06	6.16
	18	.14	.24	.40	.42	1.15	1.58	1.91	.72	1.70	2.80	3.88	4.24	1.74	3.46	5.34	7.09	.52	1.20	2.07	3.16	4.38	5.71	7.10	8.50
	20	.14	.32	.45	.50	1.36	1.88	2.25	.80	1.92	3.24	4.50	5.04	1.90	3.88	6.12	8.36	.57	1.28	2.24	3.42	4.82	6.08	8.02	9.77	11.42
DEG.	IN.	15/6	15/10	15/14	15/20	30/12	30/16	30/20	45/6	45/10	45/14	45/18	45/20	60/8	60/12	60/16	60/20	75/4	75/6	75/8	75/10	75/12	75/14	75/16	75/18	75/20

* (Inches deflection per pound) $\times 10^3$

TABLE 4

MATRIX OF INFLUENCE COEFFICIENTS*
FOR 60 DEGREE SECTOR

Radius - 20 inches
Ave. thickness - .251 inches
24 ST Aluminum Plate

DEG.	IN.																					
15	6	.02																				
	10	.01	.05																			
	14	.02	.05	.12																		
	20	.01	.05	.14	.55																	
30	12	.04	.11	.17	.18	.40																
	18	.04	.12	.26	.42	.51	.88															
	20	.04	.13	.34	.78	.59	1.23	2.08														
45	6	.06	.05	.06	.02	.18	.16	.14	.24													
	10	.08	.12	.16	.13	.45	.52	.55	.32	.68												
	14	.09	.17	.28	.34	.70	1.02	1.26	.34	.88	1.54											
	18	.09	.21	.41	.62	.90	1.55	2.17	.35	1.08	2.08	3.26										
	20	.09	.22	.46	.77	1.02	1.81	2.66	.37	1.14	2.32	3.85	4.68									
60	4	.04	.02	.02	.00	.09	.02	.06	.18	.19	.17	.16	.17	.22								
	6	.07	.06	.06	.02	.22	.20	.18	.32	.42	.43	.43	.44	.27	.49							
	8	.09	.10	.11	.06	.37	.38	.35	.41	.67	.76	.82	.85	.28	.58	.86						
	10	.10	.14	.18	.12	.55	.61	.62	.46	.90	1.17	1.33	1.40	.28	.62	1.02	1.41					
	12	.11	.18	.24	.22	.72	.88	.95	.49	1.11	1.62	1.94	2.10	.29	.64	1.12	1.64	2.15				
	14	.12	.22	.32	.32	.90	1.18	1.36	.52	1.28	2.07	2.64	2.89	.29	.68	1.19	1.79	2.46	3.12			
	16	.12	.24	.39	.44	1.04	1.50	1.83	.55	1.42	2.44	3.39	3.79	.29	.70	1.26	1.96	2.75	3.62	4.47		
	18	.13	.27	.46	.56	1.17	1.77	2.28	.56	1.55	2.84	4.15	4.72	.28	.71	1.30	2.08	3.03	4.08	5.21	6.29	
	20	.14	.30	.52	.69	1.31	2.09	2.76	.59	1.68	3.16	4.83	5.67	.30	.74	1.40	2.24	3.27	4.48	5.80	7.23	8.56
DEG.	IN.	15/6	15/10	15/14	15/20	30/12	30/16	30/20	45/6	45/10	45/14	45/18	45/20	60/4	60/6	60/8	60/10	60/12	60/14	60/16	60/18	60/20

* (Inches deflection per pound) x10^3

TABLE 5

MATRIX OF INFLUENCE COEFFICIENTS*
FOR 45 DEGREE SECTOR

Radius - 20 inches
Ave. Thickness - .251 inches
24 ST Aluminum Plate

DEG.	INS.	7.5/20	15/6	15/10	15/14	15/16	15/18	15/20	22.5 20	30/8	30/12	30/14	30/16	30/18	30/20	37.5 20	45/4	45/6	45/8	45/10	45/12	45/14	45/16	45/18	45/20
7.5	20	.15																							
15	6	.00	.02																						
	10	.00	.02	.06																					
	14	.04	.01	.06	.14																				
	16	.06	.01	.05	.15	.20																			
	18	.14	.00	.05	.15	.22	.31																		
	20	.21	.01	.04	.15	.24	.39	.57																	
22.5	20	.23	.01	.09	.27	.40	.57	.74	1.28																
30	8	.00	.05	.08	.07	.06	.05	.04	.10	.18															
	12	.04	.04	.12	.20	.20	.19	.18	.38	.21	.44														
	14	.08	.04	.13	.25	.28	.30	.30	.61	.20	.50	.68													
	16	.12	.04	.14	.31	.37	.42	.45	.90	.21	.55	.77	.98												
	18	.18	.03	.14	.34	.45	.55	.64	1.22	.20	.59	.86	1.17	1.50											
	20.	.24	.03	.15	.38	.55	.69	.82	1.59	.19	.62	.95	1.36	1.80	2.32										
37.5	20	.23	.05	.19	.47	.62	.73	.85	1.75	.28	.88	1.26	1.74	2.23	2.75	3.68									
45	4	.00	.03	.02	.02	.01	00	.00	.01	.08	.05	.05	.04	.03	.03	.05	.13								
	6	.00	.06	.06	.04	.04	.02	.02	.05	.17	.16	.14	.14	.12	.12	.18	.14	.28							
	8	.00	.07	.10	.10	.09	.07	.08	.14	.27	.31	.30	.30	.29	.27	.42	.13	.31	.50						
	10	.02	.07	.14	.17	.16	.15	.12	.29	.32	.50	.52	.54	.53	.54	.80	.12	.31	.56	.80					
	12	.04	.07	.18	.25	.28	.25	.22	.51	.35	.86	.78	.83	.87	.90	1.32	.11	.30	.57	.90	1.23				
	14	.08	.07	.20	.33	.36	.37	.35	.79	.35	.79	.99	1.15	1.26	1.35	1.95	.09	.28	.57	.96	1.42	1.84			
	16	.12	.07	.22	.41	.47	.51	.51	1.12	.36	.90	1.21	1.47	1.70	1.90	2.70	.09	.28	.57	1.01	1.55	2.11	2.67		
	18	.17	.07	.23	.48	.58	.64	.68	1.47	.37	1.00	1.36	1.76	2.12	2.46	3.50	.08	.27	.60	1.05	1.86	2.38	3.11	3.88	
	20	.22	.07	.27	.55	.69	.78	.86	1.84	.40	1.08	1.54	2.07	2.55	3.04	4.30	.08	.28	.60	1.09	1.75	2.88	3.51	4.53	5.87

* (Inches deflection per pound) x10^3

TABLE 6

MATRIX OF INFLUENCE COEFFICIENTS*
FOR 30 DEGREE SECTOR

Radius - 20 inches
Ave. Thickness - .251 inches
24 ST Aluminum Plate

DEG.	IN.																	
7.5	20	.13																
15	6	.00	.03															
	10	.00	.01	.06														
	14	.03	.00	.03	.11													
	16	.07	.00	.04	.13	.19												
	18	.13	.00	.04	.14	.23	.32											
	20	.21	.00	.02	.14	.24	.38	.58										
22.5	20	.24	.01	.07	.26	.42	.59	.78	1.43									
30	4	.00	.02	.01	.00	.00	.00	.00	.00	.07								
	6	.00	.05	.04	.01	.01	.01	.00	.02	.06	.13							
	8	.00	.04	.07	.04	.04	.04	.02	.08	.04	.12	.21						
	10	.00	.04	.10	.09	.09	.09	.06	.16	.02	.09	.20	.32					
	12	.02	.02	.10	.17	.18	.17	.14	.34	.01	.06	.16	.34	.49				
	14	.06	.02	.12	.24	.28	.30	.27	.59	.01	.06	.16	.32	.54	.75			
	16	.11	.02	.10	.29	.38	.44	.45	.95	.00	.04	.13	.30	.54	.84	1.11		
	18	.18	.02	.11	.33	.47	.59	.67	1.35	.01	.04	.12	.28	.53	.89	1.32	1.72	
	20	.25	.01	.10	.38	.54	.73	.90	1.79	.00	.03	.11	.27	.54	.93	1.46	2.05	2.72
DEG.	IN.	75/20	15/6	15/10	15/14	15/16	15/18	15/20	22.5 20	30/4	30/6	30/8	30/10	30/12	30/14	30/16	30/18	30/20

* (Inches deflection per pound) x10^3

TABLE 7

		i = 45° / 12"		i = 45° / 16"		i = 45° / 18"	
S	j	$_vg_{ij}$	$_mg_{ij}$	$_vg_{ij}$	$_mg_{ij}$	$_vg_{ij}$	$_mg_{ij}$
1.0	1	12.20	-.39	24.20	-1.29	32.50	-2.24
1.0	2	10.95	-.33	22.10	-1.17	29.65	-2.08
1.0	3	9.80	-.28	19.90	-1.06	26.75	-1.88
1.0	4	8.60	-.22	17.60	- .94	23.80	-1.66
1.0	5	7.32	-.17	15.35	- .80	20.80	-1.43
1.0	6	6.15	-.11	13.10	- .63	17.75	-1.20
1.0	7	5.10	-.07	10.90	- .48	14.85	- .98
1.0	8	4.10	-.02	8.95	- .36	12.10	- .75
1.0	9	3.15	+.02	7.00	- .24	9.50	- .53
1.0	10	2.30	+.07	5.20	- .12	7.22	- .35
1.0	11	1.60	+.10	3.65	+ .01	5.18	- .18
1.0	12	1.00	+.11	2.40	+ .08	3.40	- .02
1.0	13	.60	+.10	1.50	+ .12	2.05	+ .08
1.0	14	.15	+.09	.62	+ .13	.90	+ .11
1.0	15	.01	+.05	.18	+ .10	.29	+ .10
0.7	16	.00	+.02	.00	+ .05	.01	+ .05
		$_cg_{ij}$ = 12.57 j = 45° / 20"		$_cg_{ij}$ = 25.15 j = 45° / 20"		$_cg_{ij}$ = 33.55 j = 45° / 20"	

Table 8a

Influence Coefficients

$_vg_{ij} = \left[\text{Inches Deflection/Pound}\right] 10^3$

$_mg_{ij} = \left[\text{Inches Deflection/Inch Pound}\right] 10^3$

$_cg_{ij} = \left[\text{Inches Deflection/Pound}\right] 10^3$

Sector Angle = 45°, t = 1/8 Inches, Radius = 20 Inches

Material—24 ST Aluminum Alloy

$$W_i = \Big[\sum_{j=1}^{15} V_{(j)} \cdot {}_vg_{ij} + .71 V_{i(16)}\, {}_vg_{i(16)} + \sum_{j=1}^{15} M_{(j)}\, {}_mg_{ij} + \cdots$$

$$+ .7' M_{(16)}\, {}_mg_{i(16)} + P\, {}_cg_{ij} \Big] 10^3$$

		$i = 45^\circ/20''$		$i = 30^\circ/20''$		$i = 40^\circ/20''$	
S	j	${}_v g_{ij}$	${}_m g_{ij}$	${}_v g_{ij}$	${}_m g_{ij}$	${}_v g_{ij}$	${}_m g_{ij}$
1.0	1	40.25	-3.92	35.00	-3.42	22.90	-2.24
1.0	2	36.70	-3.48	32.70	-3.27	22.15	-2.20
1.0	3	33.05	-3.02	30.10	-2.98	21.25	-2.17
1.0	4	29.37	-2.58	27.25	-2.62	20.25	-2.16
1.0	5	25.70	-2.14	24.20	-2.23	19.05	-2.16
1.0	6	22.05	-1.76	21.10	-1.87	17.40	-2.07
1.0	7	18.60	-1.40	18.00	-1.54	15.40	-1.75
1.0	8	15.35	-1.07	14.95	-1.22	13.20	-1.44
1.0	9	12.05	- .78	11.90	- .92	10.90	-1.15
1.0	10	9.00	- .51	9.00	- .64	8.60	- .80
1.0	11	6.42	- .28	6.50	- .40	6.45	- .61
1.0	12	4.30	- .10	4.50	- .18	4.50	- .34
1.0	13	2.50	+ .04	2.90	+ .00	2.60	- .11
1.0	14	1.25	+ .12	1.20	+ .10	1.20	+ .02
1.0	15	.40	+ .12	.35	+ .10	.35	+ .08
0.7	16	.01	+ .05	.01	+ .04	.01	+ .07

${}_c g_{ij} = 42.05$	${}_c g_{ij} = 35.88$	${}_c g_{ij} = 23.15$
$j = 45^\circ / 20''$	$j = 45^\circ / 20''$	$j = 45^\circ / 20''$

Table 8b

Influence Coefficients

${}_v g_{ij} = \left[\text{Inches Deflection/Pound}\right] 10^3$

${}_m g_{ij} = \left[\text{Inches Deflection/Inch Pound}\right] 10^3$

${}_c g_{ij} = \left[\text{Inches Deflection/Pound}\right] 10^3$

Sector Angle = 45°/20", t = 1/8 Inches, Radius = 20 Inches

$$W_{ij} = \left[\sum_{j=1}^{15} V_{(j)}\, {}_v g_{ij} + .7\, V_{(16)}\, {}_v g_{i(16)} + \sum_{j=1}^{15} M_{(j)}\, {}_m g_{ij} + \cdots \right.$$

$$\left. + .7\, M_{(16)}\, {}_m g_{i(16)} + P\, {}_c g_{ij} \right] \times 10^3$$

		Load at				
		45°/16"	45°/18"	45°/20"	30°/20"	15°/20"
Meter at	45°/16"	7.30*				
	45°/18"	7.34	7.35			
	45°/20"	7.26	7.30	7.39		
	30°/20"	7.23	7.34	7.29	7.31	
	15°/20"	7.29	7.32	7.16	7.19	7.26

* $\frac{\text{Influence Coefficients .125 inch Sector}}{\text{Influence Coefficient .251 inch Sector}}$

TABLE 9

Effect of Thickness

on Stiffness

Deg./In.	A_{180°
15/6	5.5850
15/10	10.4720
15/14	14.6608
15/20	0.0000
30/12	19.7804
30/16	21.4672
30/20	14.1368
45/6	5.5850
45/10	10.4720
45/14	14.6608
45/18	18.8491
45/20	0.0000
60/8	13.0550
60/12	12.5664
60/16	16.7550
60/20	9.4246
75/10	10.4720
75/14	14.6608
75/18	18.8491
75/20	0.0000
90/4	5.4454
90/8	10.6814
90/12	12.5664
90/16	16.7550
90/20	9.4246
105/6	6.2832
105/10	10.4720
105/14	14.6608
105/18	18.8491
120/8	13.4041
120/12	12.5664
120/16	16.7550
120/20	9.4246
135/8	10.4720
135/12	14.6608
135/16	18.8491
135/20	0.0000
150/4	5.4454
150/8	10.6814
150/12	12.5664
150/16	16.7550
150/20	9.4246
165/6	6.2832
165/10	10.4720
165/14	14.6608
165/18	18.8491
165/20	0.0000
180/12	10.1230
180/14	0.0000
180/16	8.3775
180/18	0.0000
180/20	4.7123

Deg./In.	A_{135°
15/6	5.5850
15/10	10.4720
15/14	14.6608
15/20	0.0000
30/12	19.7804
30/16	21.4672
30/20	14.1368
45/6	5.5850
45/10	10.4720
45/14	14.6608
45/18	18.8491
45/20	0.0000
60/8	13.0550
60/12	12.5664
60/16	16.7550
60/20	9.4246
75/10	10.4720
75/14	14.6608
75/18	18.8491
75/20	0.0000
90/4	5.4454
90/8	10.6814
90/12	12.5664
90/16	16.7550
90/20	9.4246
105/6	6.2832
105/10	10.4720
105/14	14.6608
105/18	18.8491
120/8	10.8909
120/12	12.5664
120/16	16.7550
120/20	9.4246
135/4	1.6755
135/6	0.0000
135/8	4.2935
135/10	0.0000
135/12	6.2832
135/14	0.0000
135/16	8.3775
135/18	0.0000
135/20	4.7123

Deg./In.	A_{90°
15/6	5.5850
15/10	10.4720
15/14	14.6608
15/20	0.0000
30/12	19.7804
30/16	21.4672
30/20	14.1368
45/6	5.5850
45/10	10.4720
45/14	14.6608
45/18	18.8491
45/20	0.0000
60/8	13.0550
60/12	12.5664
60/16	16.7550
60/20	9.4246
75/10	10.4720
75/14	14.6608
75/18	18.8491
75/20	0.0000
90/4	3.3510
90/6	0.0000
90/8	5.9690
90/10	0.0000
90/12	6.2832
90/14	0.0000
90/16	8.3775
90/18	0.0000
90/20	4.1723

TABLE 10

COLUMN MATRICES OF AREA COEFFICIENTS*
FOR
SECTORS OF 20 INCH RADIUS

*INCHES^2 / POINT

Deg.	In.	A_{75°	Deg.	In.	A_{60°	Deg.	In.	A_{45°
15	6	5.5850	15	6	5.5850	7.5	20	1.9897
15	10	10.4720	15	10	10.4720	15	6	8.1214
15	14	14.6608	15	14	14.6608	15	10	10.2427
15	20	0.0000	20	20	0.0000	15	14	10.4720
30	12	19.7804	30	12	19.7804	15	16	8.3776
30	16	21.4672	30	16	21.4672	15	18	8.4300
30	20	14.1368	30	20	14.1368	15	20	3.9794
45	6	5.5850	45	6	5.5850	22.5	20	1.9897
45	10	10.4720	45	10	10.4720	30	8	7.3396
45	14	14.6608	45	14	14.6608	30	12	8.9010
45	18	18.8491	45	18	18.8491	30	14	7.3304
45	20	0.0000	45	20	0.0000	30	16	8.3776
60	8	10.5418	60	4	0.0000	30	18	8.4300
60	12	12.5664	60	6	0.0000	30	20	3.9794
60	16	16.7550	60	8	5.4105	37.5	20	1.9897
60	20	9.4246	60	10	0.0000	45	4	3.1027
75	14	1.6755	60	12	6.2832	45	6	2.9125
75	6	0.0000	60	14	0.0000	45	8	2.0944
75	8	4.2936	60	16	8.3775	45	10	2.6170
75	10	0.0000	60	18	0.0000	45	12	3.1416
75	12	6.2832	60	20	4.7123	45	14	3.6652
75	14	0.0000				45	16	4.1888
75	16	8.3775				45	18	4.2150
75	18	0.0000				45	20	1.9897
75	20	4.7123						

TABLE 11

Column Matrices of Area Coefficients*

for

Sectors of 20 Inch Radius

*Inches^2 / Point

FIGURES

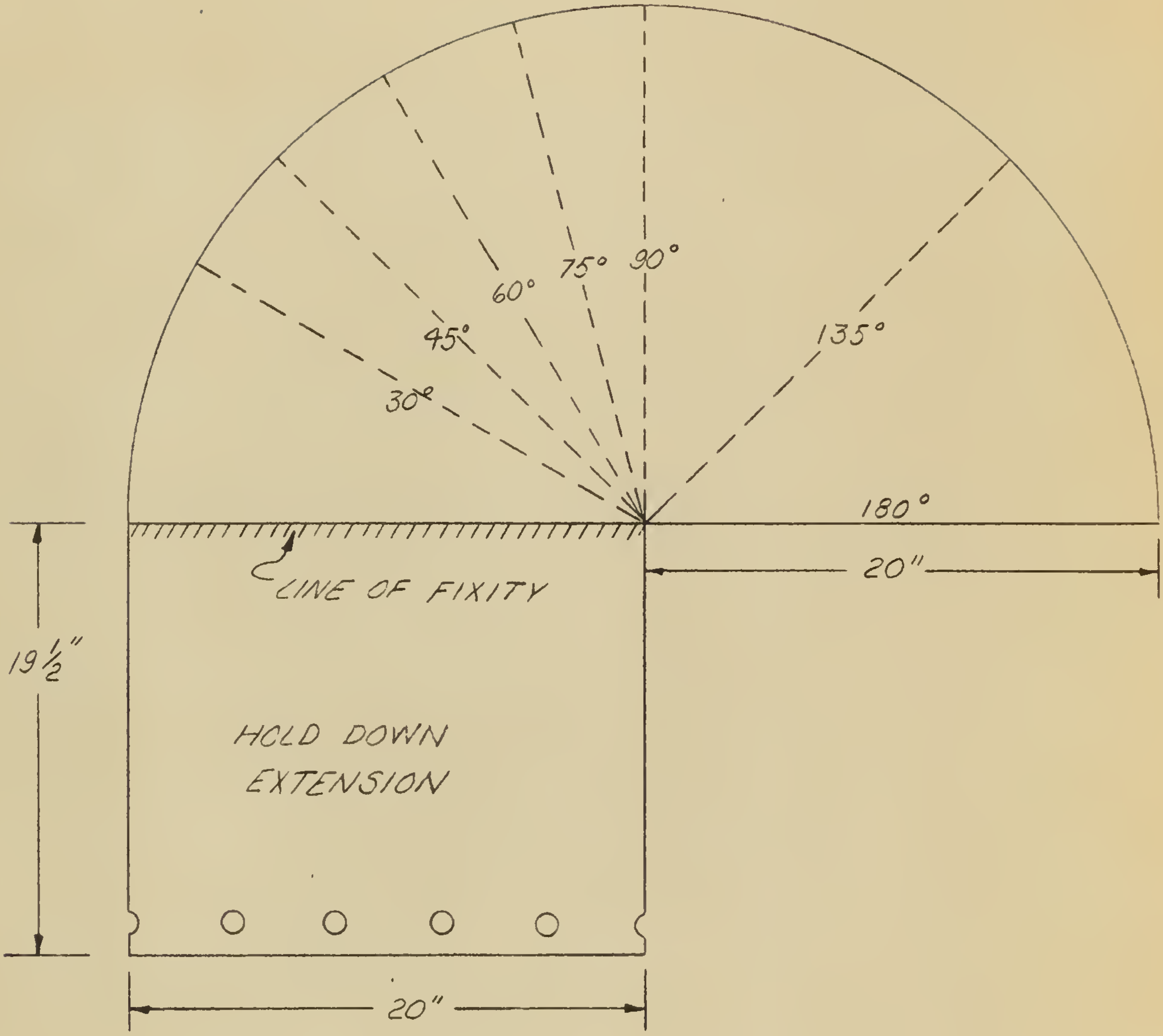


FIG. 1

PLAN FORM OF PHASE I SPECIMEN
1/4" 24ST ALUMINUM ALLOY

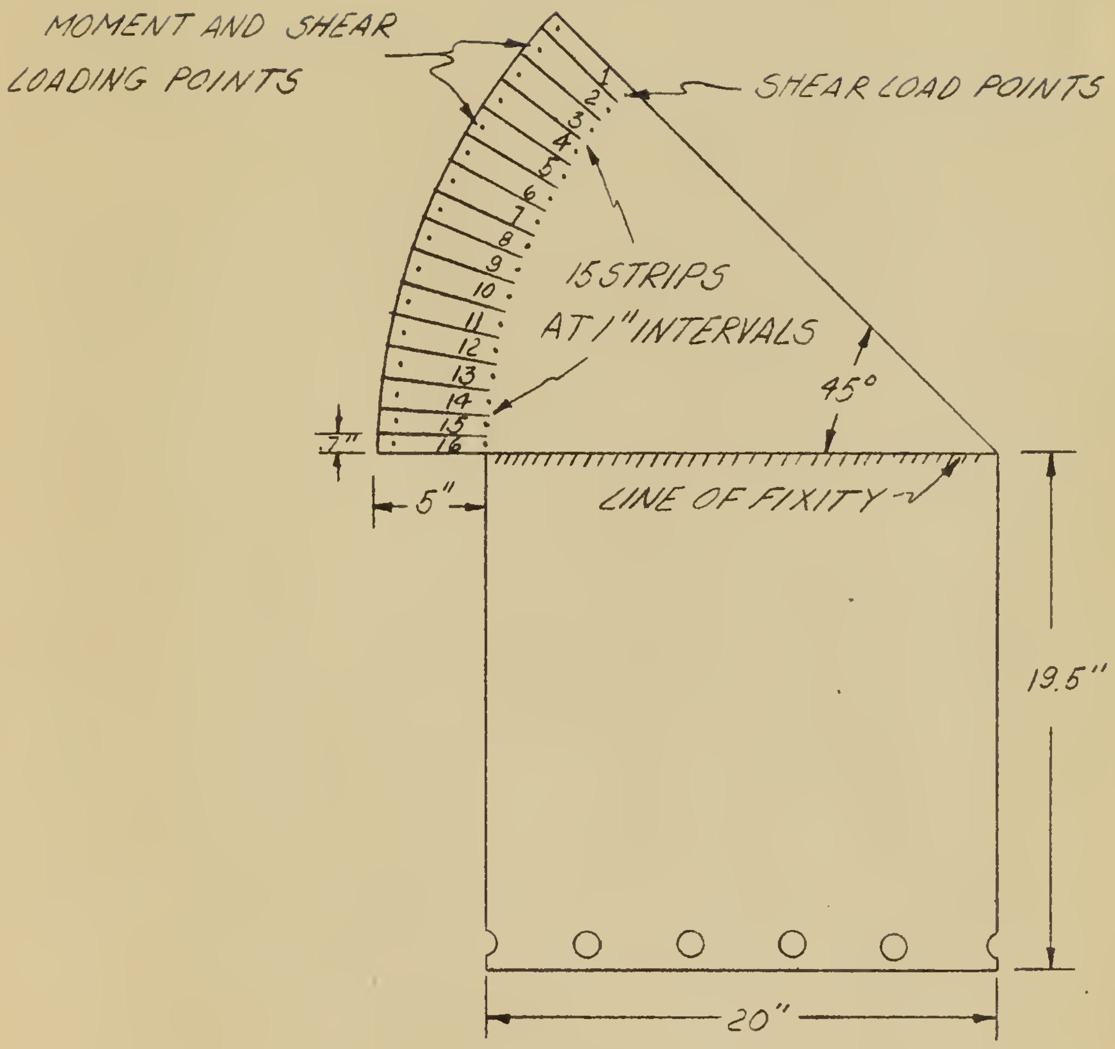


FIG. 2

PLAN FORM OF PHASE 2 SPECIMEN
1/8" 24ST ALUMINUM ALLOY

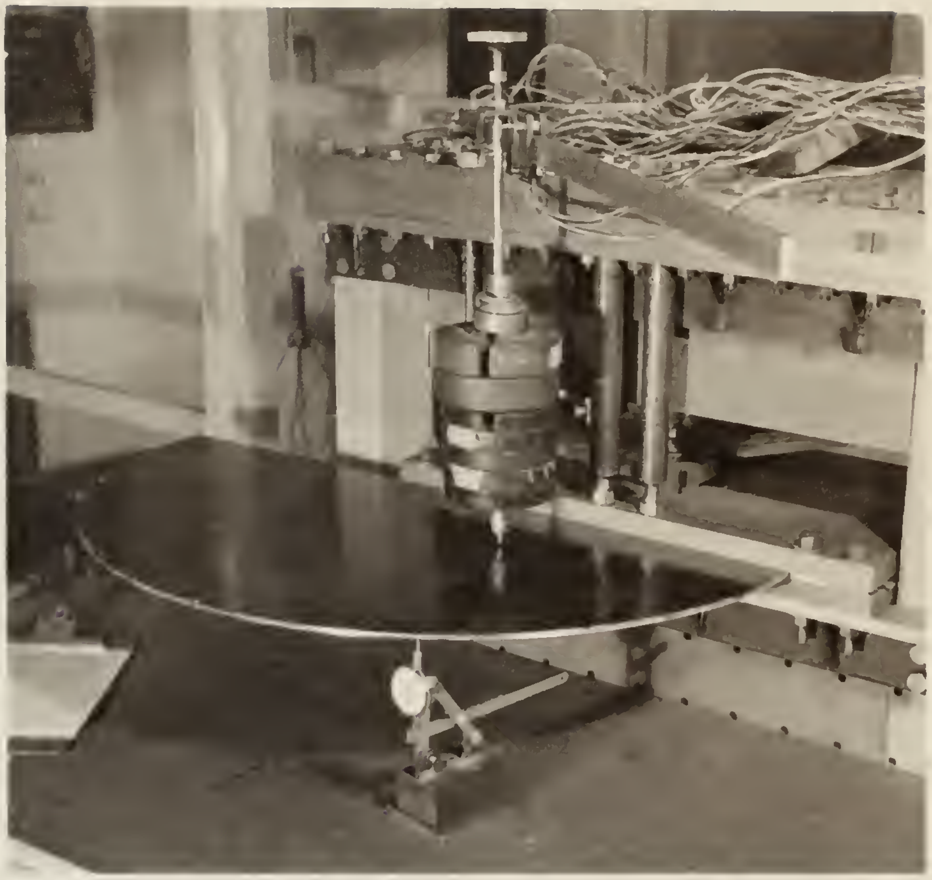


Figure 3

Arrangement for securing specimen

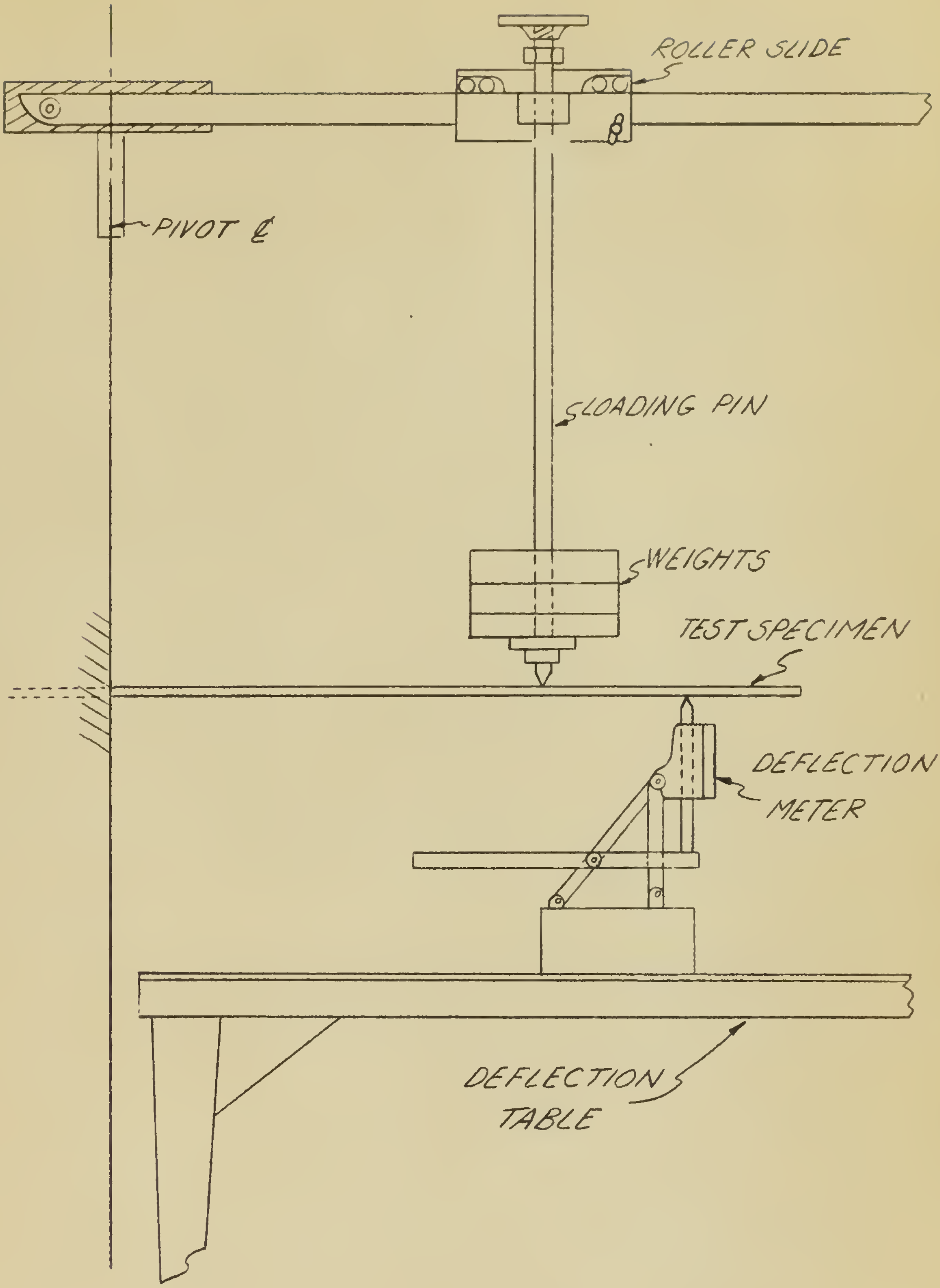


FIG. 4

ARRANGEMENT FOR LOADING AND MEASURING DEFLECTIONS OF TEST SPECIMEN

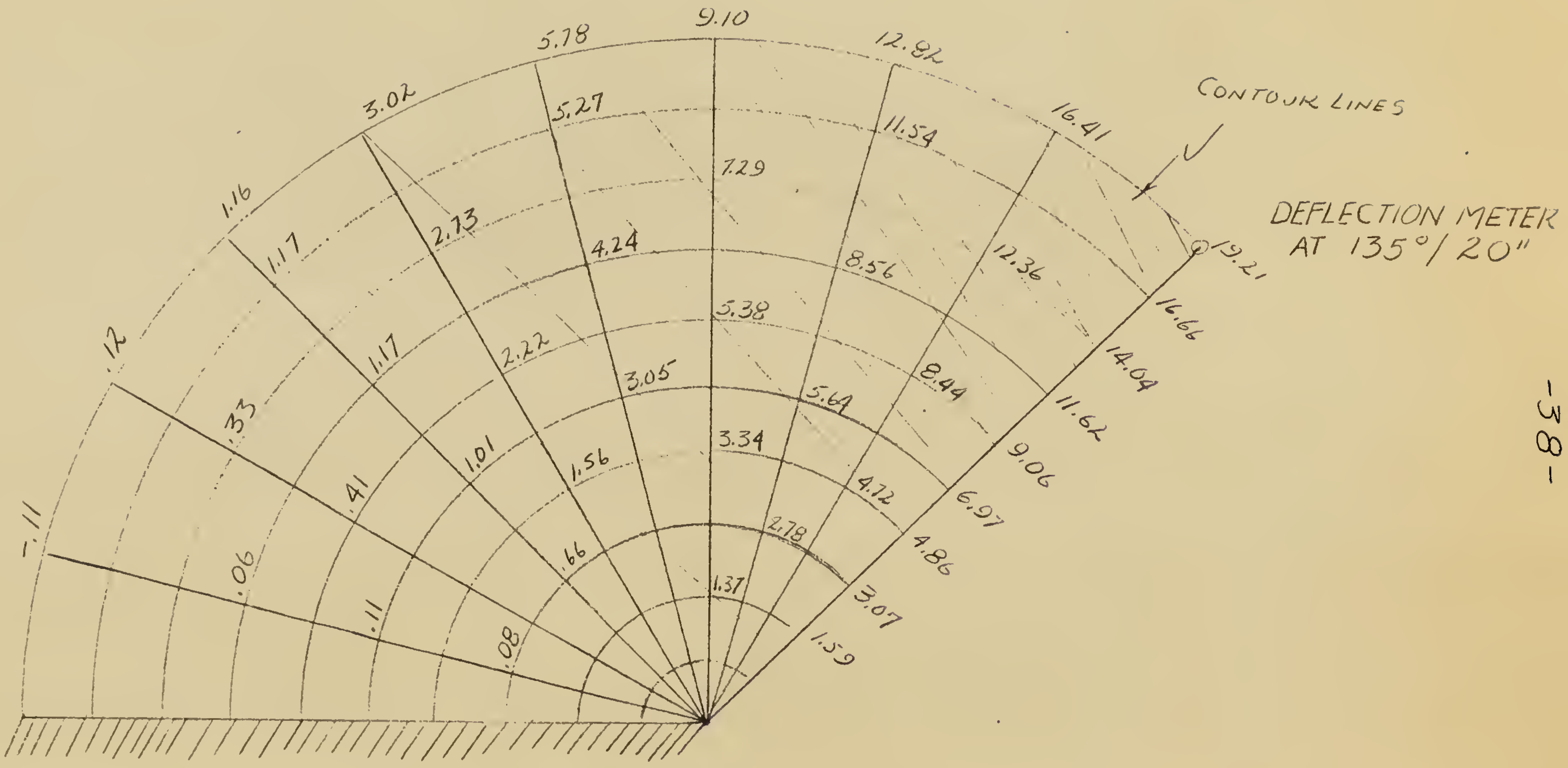


FIG. 5

SAMPLE DATA SHEET

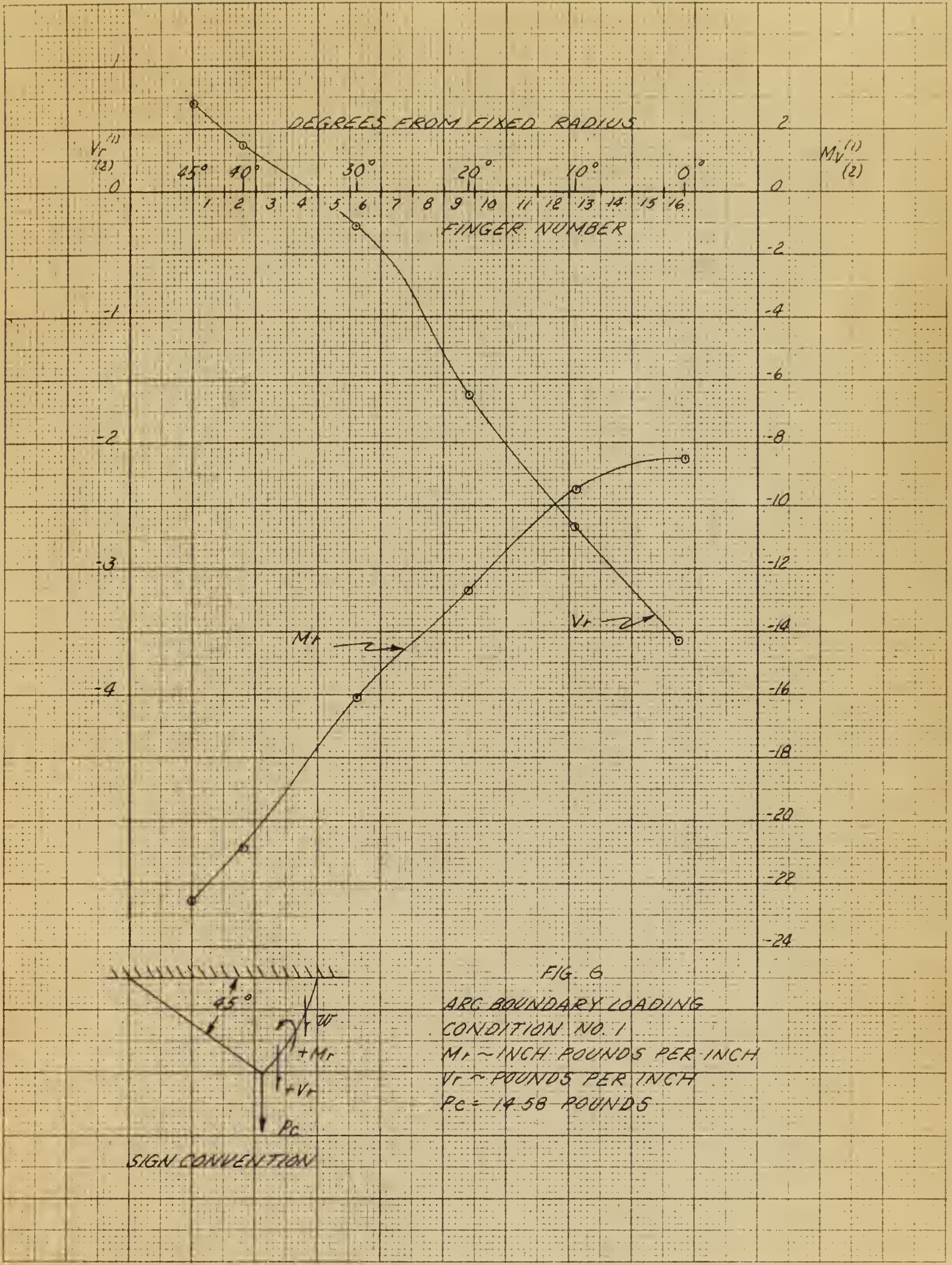


FIG. 6

ARC BOUNDARY LOADING

CONDITION NO. 1

Mr ~ INCH POUNDS PER INCH

Vr ~ POUNDS PER INCH

Pc = 14.58 POUNDS

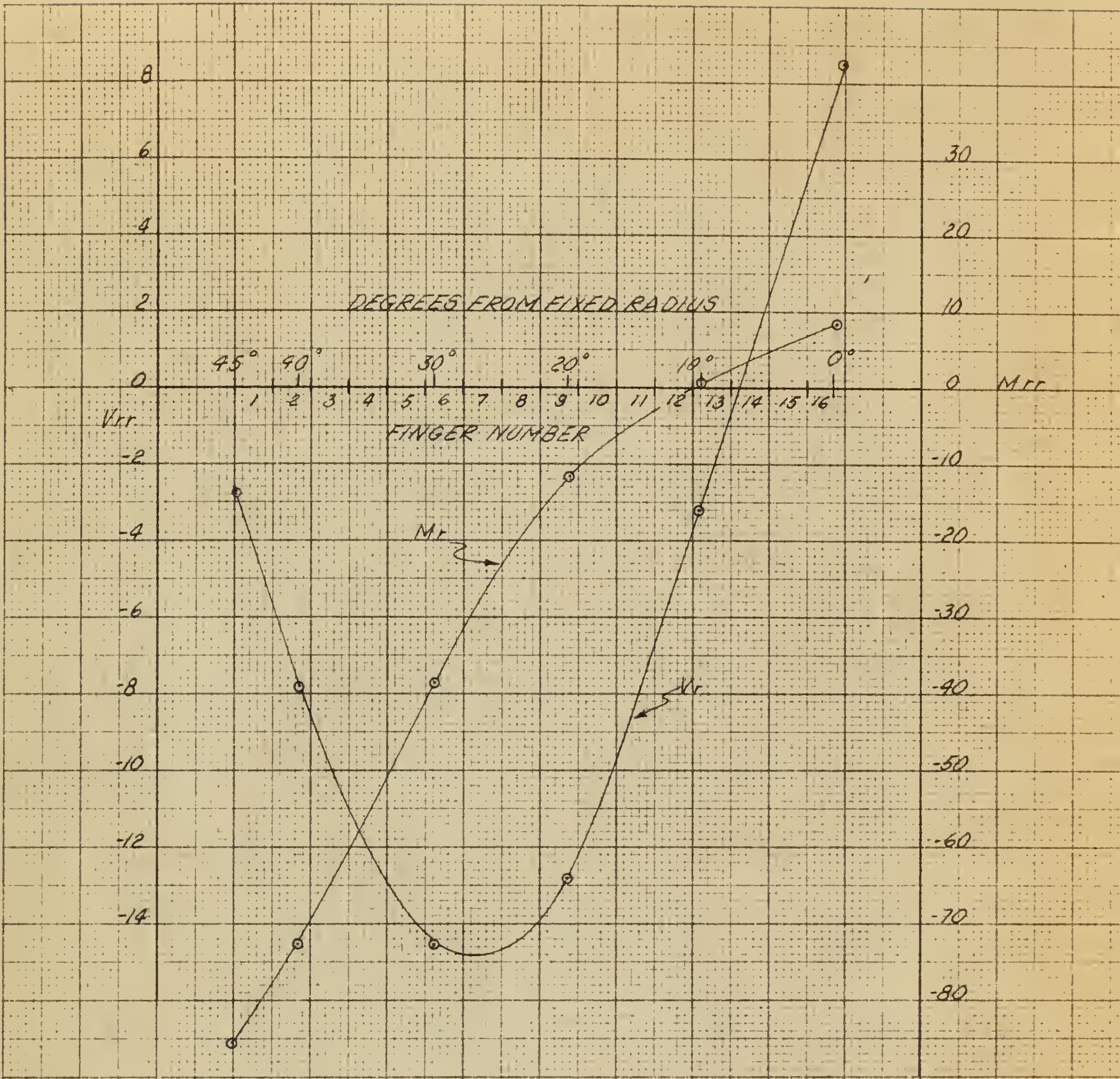
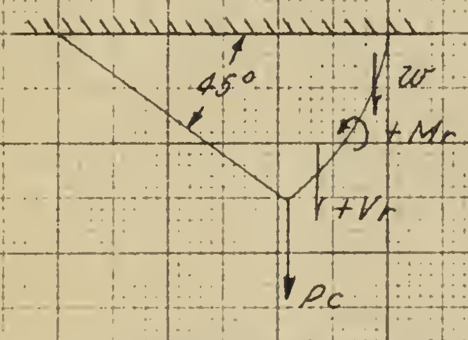


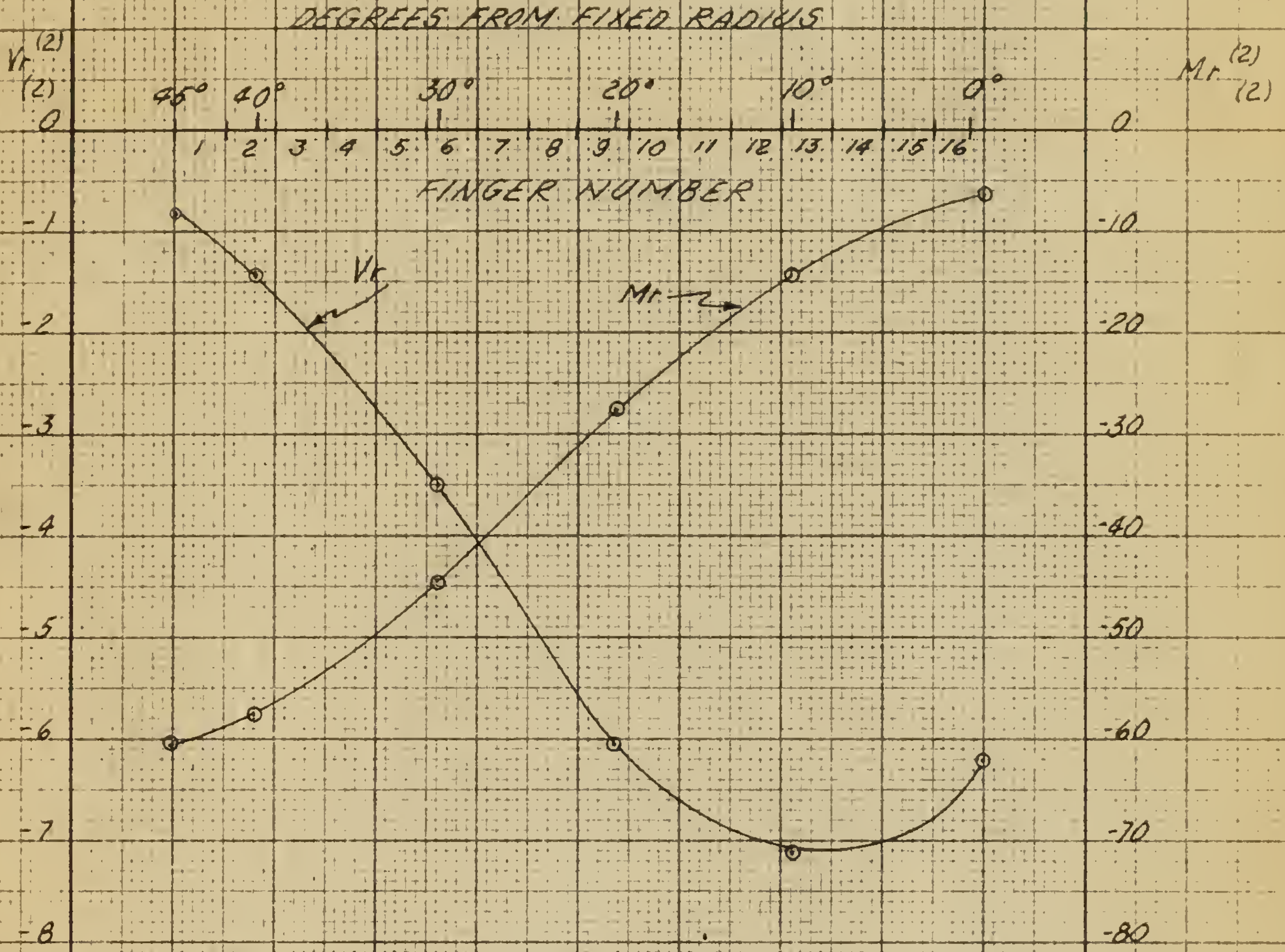
FIG. 7

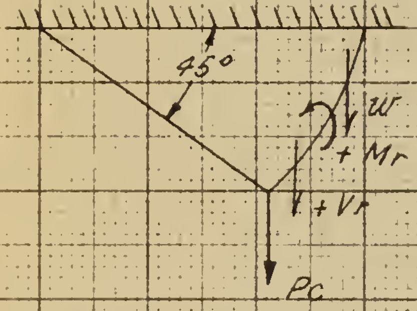
ARC BOUNDARY LOADING
CONDITION NO. 3

M_r ~ INCH POUNDS PER INCH
V_r ~ POUNDS PER INCH
P_{c_r} = 55.34 POUNDS



SIGN CONVENTION





SIGN CONVENTION

FIG. 5

ARC BOUNDARY LOADING
CONDITION NO. 2
Mr ~ INCH POUNDS PER INCH
Vr ~ POUNDS PER INCH
P_c = 13.79 POUNDS

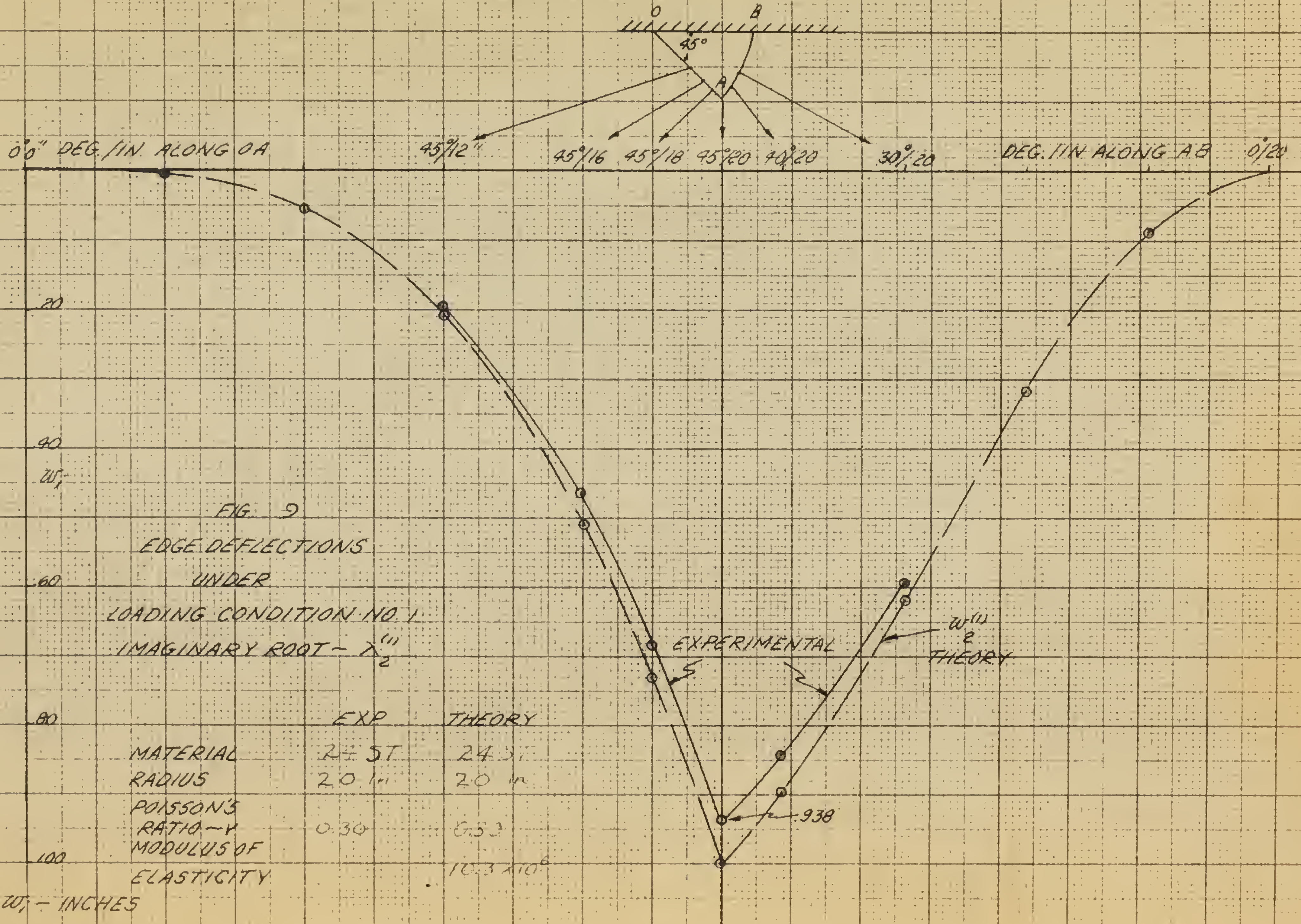


FIG. 9
EDGE DEFLECTIONS
UNDER
LOADING CONDITION NO. 1
IMAGINARY ROOT - $\lambda_2^{(1)}$

	EXP	THEORY
MATERIAL	24 ST	24 ST
RADIUS	20 in	20 in
POISSON'S RATIO - ν	0.30	0.33
MODULUS OF ELASTICITY		10.3×10^6

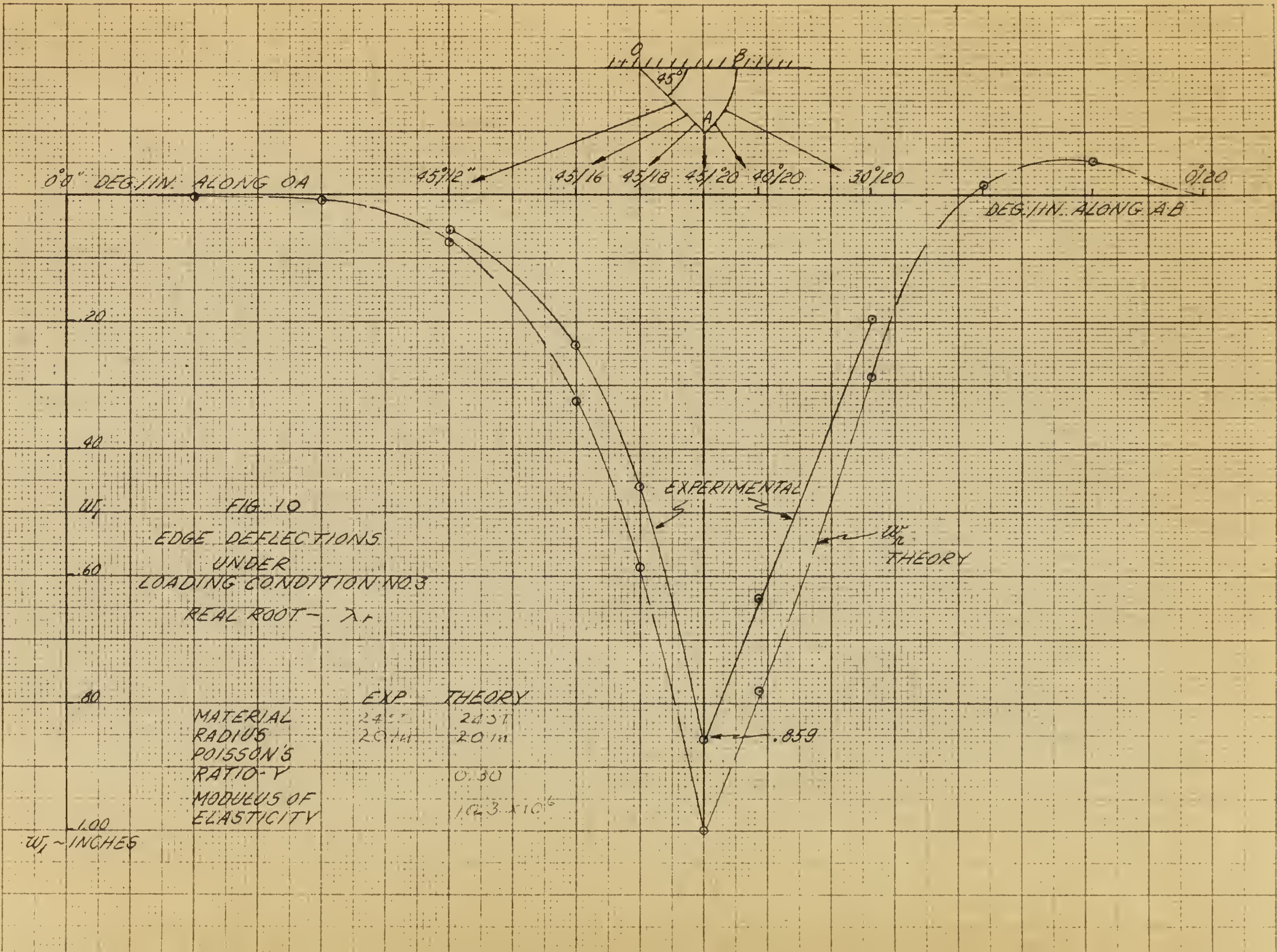


FIG. 10
EDGE DEFLECTIONS
UNDER
LOADING CONDITION NO.3
REAL ROOT – λ_r

	EXP	THEORY
MATERIAL	24ST	24ST
RADIUS	20 in	20 in
POISSON'S RATIO – ν		0.30
MODULUS OF ELASTICITY		10.3×10^6

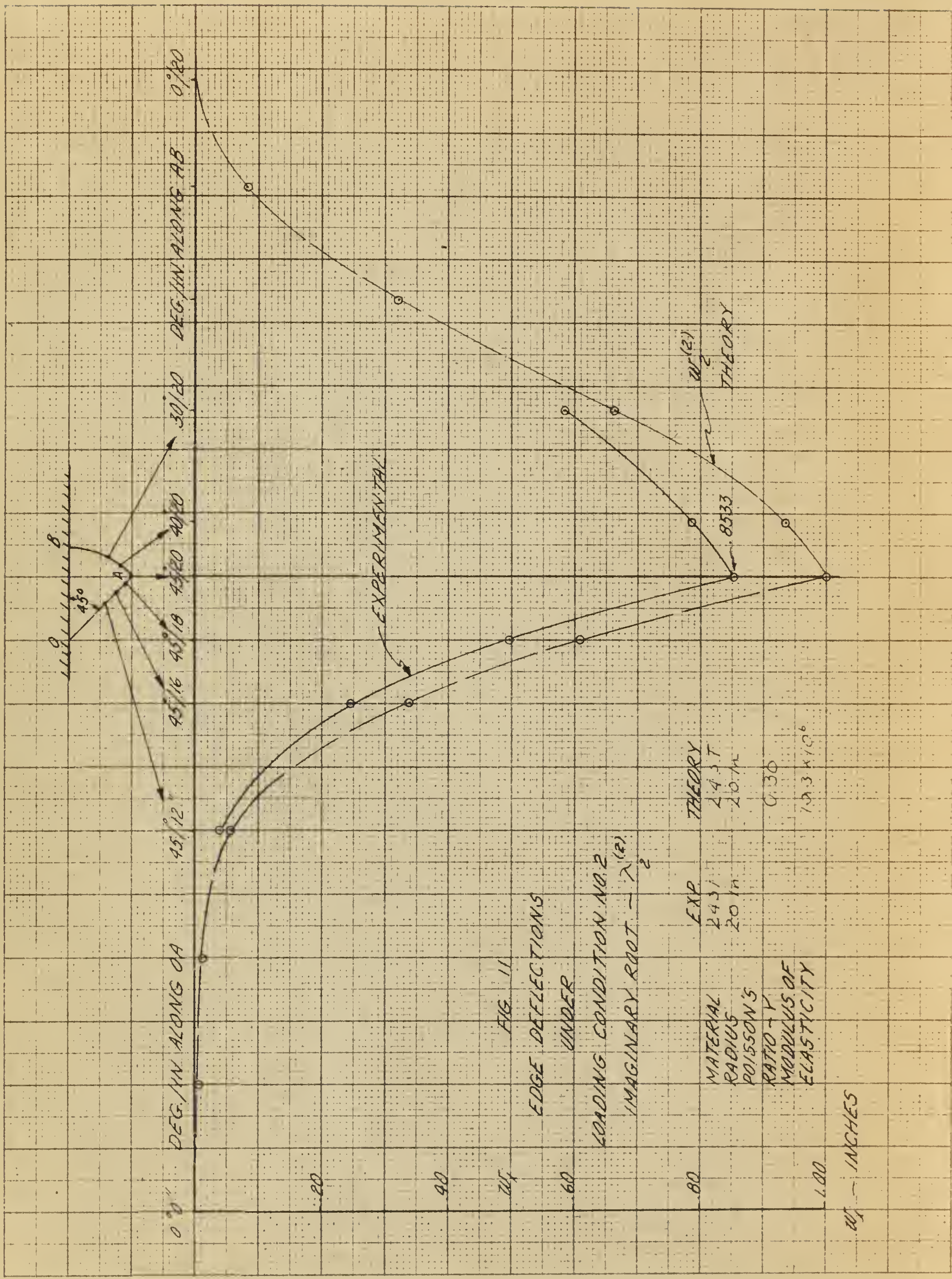


FIG. 11

EDGE DEFLECTIONS UNDER LOADING CONDITION NO. 2

IMAGINARY ROOT — $\lambda_2^{(2)}$

	EXP	THEORY
MATERIAL	24ST	24ST
RADIUS	20 in	20 in
POISSON'S RATIO – ν	0.30	
MODULUS OF ELASTICITY	10.3×10⁶	

APPENDIX

SAMPLE CALCULATIONS

Influence Coefficients

1. Concentrated load.

w_i = Deflection in inches at "i".

P_j = Concentrated load in pounds at "j".

g_{ij} = Influence coefficient, inches/1000 pounds.

= $\left[\text{Inches deflection at "i" per pound at "j"}\right] \times 10^3$.

$w_{ij} = P_j g_{ij} \times 10^{-3}$ 1.1

$g_{ij} = 20\ w_{ij}$ (Phase 1) 1.2

$g_{ij} = 100\ w_{ij}$ (Phase 2) 1.3

2. Shear loading along curved boundary.

w_i = Deflection in inches at "i".

V_j = Shear in pounds/inch near "j".

${}_v g_{ij}$ = Influence coefficient in inches/1000 pounds.

= $\left[\text{Inches deflection at "i" per pound near "j"}\right] \times 10^3$.

s = Boundary length of element near "j".

$w_{ij} = s\left[V_j\ {}_v g_{ij}\right] \times 10^{-3}$ 2.1

For $V_j = 10$ pounds/inch

$s = 1$ inch

$w_{ij} = {}_v g_{ij} \times 10^{-2}$ 2.2

Or ${}_v g_{ij} = w_{ij} \times 10^2$

It can be seen that the influence coefficient is independent of s, so the strip number 16 is covered by (2.2). $S = 71$ inches for strip 16).

By superposition of deflections

$$ {}_{v}w_{ij} = \sum_{j=1}^{15} V_j \, {}_{v}g_{ij} \times 10^{-3} + .71 \, V_{16} \, {}_{v}g_{i16} \times 10^{-3} $$

3. Radial moments along curved boundaries.

w_i = Deflection in inches at "i".

M_j = Radial moment in inch pounds/inch near "j".

= 49.9 inch pounds/inch. (Phase 2)

${}_{M}g_{ij}$ = Influence coefficient in inches deflection per 1000 inch pounds.

= [Inches deflection at "i" per pound near "j"] x 10^3.

s = Boundary length of element near "j".

For a 10.4 pound load 4.8 inches from the root of the radial strips the deflection is determined from:

$$ w_{ij} = s(V_j \, {}_{v}g_{ij} + M \, {}_{m}g_{ij}) \times 10^{-3} \qquad 3.1 $$

For the strip with a .71 inch root

$$ w_{ij} = .71\left(\frac{10.4}{.71} \, {}_{v}g_{ij} + \frac{49.9}{.71} \, {}_{m}g_{ij} \right) \times 10^{-3} $$

From the above equation

$$ {}_{m}g_{ij} = \frac{w_{ij} \times 10^{3} - 10.4 \, {}_{v}g_{ij}}{49.9} \qquad 3.2 $$

The following is a discussion of the problem of determining the deflection pattern of sectors with radii, thickness, and material constants different from those of the sectors used in this investigation.

The loading and deflection points on the new sector are located in the same geometric position as they are on the sector used in this investigation. The grid on the new sector will be 15 degrees by $\frac{r}{r_0}$. Where "r_0" is the radius of the sector and "r" is the radius of the loading or deflection point. The value of $\frac{r}{r_0}$ for the new sector is equal to $\frac{r}{r_0}$ for the corresponding sector in this investigation.

The deflection of a sector varies inversely as the plate stiffness (D) and directly as some power of the radius.

The use of dimensional analysis will give the ratio of the loadings and the radius necessary to use with the deflection data presented and produce accurate results.

Elasticity relationships may be used with the data presented in this investigation to approximate deflection patterns of sectors with radii, thickness, and material constants different from the sectors surveyed in this investigation if the deflection mode is expressed as a function of the radius.

The deflection mode for loading conditions used in this investigation can be obtained from the data presented in the tables. Using this deflection mode an expression of the deflection in terms of the radius can be obtained. The deflection of the new sector becomes:

$$w' = w\frac{D}{D'} \; f\left(\frac{r_0'}{r_0}\right)^n \qquad\qquad D = \frac{Et^3}{12(1-\nu^2)}$$

The accuracy of the deflection (w') depends on how closely the deflection mode of the sector used in this investigation can be approximated.

It is left to the reader to determine and develop the method used in the deflection survey of any sector of different radius, thickness, and material constants from those of the sectors in this investigation.

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